Space-Time Matrix Method for Joint DOA and Delay Estimation

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ABSTRACT

High-resolution parameter estimation techniques have recently been applied to jointly estimate multipath signal parameters. In this paper, we consider the problem of estimating the directions of arrival (DOAs) and propagation delays of multipath narrowband signals in wireless communications. We define a novel space-time matrix which includes the information of DOAs and delays of multipath signals. The joint DOA and delay estimation method used the defined space-time matrix is named as space-time matrix method. Making using of the proposed method, the DOAs and delays can be estimated by the eigenvalues and corresponding eigenvectors of the defined space-time matrix, respectively. So, the presented method can automatically determined the paring of the estimated DOAs and delays by the relationship between eigenvalues and eigenvectors. Compared with the previous work, the space-time matrix method can provide better performance with substantially reduced computational complexity. The performances of some relevant algorithms are compared via simulations, and the effectiveness of the proposed method is verified.

Keywords: Array signal processing, direction of arrival, delay, joint estimation, multipath

1. INTRODUCTION

In wireless communications, one interesting problem is to try to estimate the angles of incidence and path delays of emitted signals arriving at a base station antenna array, namely, the JADE problem. It finds applications in source localization, accident reporting, and intelligent transportation. Furthermore, in a multipath wireless communication system, one can obtain a better channel estimate by jointly exploring the ray DOAs and the ray propagation delays, thus significantly improving the system performance.

Recently, various high resolution methods have been proposed to jointly estimate DOAs and delays of multipath signals. These methods can be classified into three categories [1]: spectral estimation, parametric subspace-based estimation (PSBE), and deterministic parametric estimation (DPE). SAGE [2] and ML [3] algorithms are the worth mentioning two within the third category. The third category is a high dimension nonlinear optimization procedure, its computational complexity is extremely demanding. The first and the second categories can achieve simpler and suboptimal solutions, but they rely on the eigendecomposition (EVD) of the observation space into signal subspace and noise subspace. For example, in spectral estimation, Vanderveen et al. proposed the JADE-MUSIC [4] algorithm, which exploited the properties of the space-time structure by stacking the received data. After performing a high-dimensional EVD on the covariance matrix, the channel parameters can be estimated by a 2-D searching on the DOA-delay plane. The required computations, however, make JADE-MUSIC also unfavorable for real-time implementation. To lower the computational overhead, Yung-Yi Wang et al. presented the TST-MUSIC [5, 6] algorithm, which combines the techniques of temporal filtering and of spatial beamforming with three one-dimensional (1-D) MUSIC algorithm, i.e., one S-MUSIC and two T-MUSIC algorithms. Compared with the JADE-MUSIC, the TST-MUSIC is low in computational complexity but produces considerably fewer estimation errors in a highly contaminated environment. Clearly, the first category algorithms need not only EVD but also 2-D search or multiple 1-D searches. In PSBE methods, Vanderveen et al. presented JADE-ESPRIT [7] and SI-JADE [9] algorithms, which utilize shift invariance property of the estimated channel matrix and don't need searching process. These algorithms directly use eigenvalues to estimate the parameters. However, vectorization (column stacking), Khatri-Rao product and Kronecker product etc increase the dimension of the data matrices that need EVD, which negatively affect real-time processing. To decrease the computational complexity of the JADE-ESPRIT and SI-JADE algorithms, Yung-Yi Wang et al. presented the TST-ESPRIT [9] algorithm which employs three 1-D ESPRIT-type algorithms, i.e. two T-ESPRITs and one S-ESPRIT.

In this paper, we propose a lower computational complexity algorithm to jointly estimate DOA and delay by defining a space-time matrix. The proposed

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approach possesses several attractive features. First, compared with the previous work, the matrices that need EVD are smaller-sized matrices. Second, it utilizes the eigenvalues and corresponding eigenvectors of the defined space-time matrix to estimate DOAs and delays, respectively. Finally, the paring of the estimated DOAs and delays is automatically determined by the relationship of between eigenvalue and eigenvector. The outline of the paper is organized as follows. Section II briefly introduces the system model of the fading multiray channels, which assume the propagation rays to be specular rather than dispersed. The space-time matrix method for joint DOA and delay estimation is developed in Section III. In Section IV, we analyze the theoretical performance of the proposed algorithm. The issue of computational complexity is addressed as well. In Section V, simulation results are presented to verify the performance of the proposed approach. Section VI provides a concluding remark to summarize the paper

2. DATA MODEL

In this paper, we consider a TDMA system with a known training sequence embedded in each data burst. The radio channel in a wireless communication system is often characterized by a multiray propagation model. Assume a uniform linear array (ULA) with M isotropic sensors spaced by the distance d. The training signals received at the antenna array, during the nth burst can be expressed as follows [5]:

$$\mathbf{x}^{(n)}(\mathbf{t}) = \sum_{i=1}^{Q} \mathbf{a}(\theta_i)\beta_i(\mathbf{n})\tilde{\mathbf{s}}(\mathbf{t}-\tau_i) + \mathbf{n}(\mathbf{t}) \qquad (1)$$

where $\mathbf{x}^{(n)}(t)$ is received signals in the nth time burst; $\mathbf{n}(t)$ represents the spatially and temporally white additive Gaussian noises with zero-means and equal variance σ_n^2; $\mathbf{a}(\theta_i)$ denotes steering vector of a signal arriving from direction θ_i; $\beta_i(n)$ stands for ray amplitude that is a complex Gaussian random process; $\tilde{s}(\cdot)$ is transmitted complex signal; τ_i denotes propagation delay of the ith ray; Q is total number of rays present in the system.

In matrix form, the received signals corresponding to the nth burst can be written as follows

$$\mathbf{X}^{(n)} = \underbrace{\mathbf{A}(\theta)}_{M\times Q}\underbrace{\mathbf{B}(n)}_{Q\times Q}\underbrace{\tilde{\mathbf{G}}(\tau)^T}_{Q\times N_t} + \underbrace{\mathbf{N}^{(n)}}_{M\times N_t} \quad (n=1,\cdots,K) \qquad (2)$$

where the superscript $(\cdot)^T$ represents the matrix transpose operation; $\mathbf{X}^{(n)} = [\mathbf{x}^{(n)}(1),\cdots,\mathbf{x}^{(n)}(N_t)]^T$, N_t is the length of the extracted training sequence. $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1),\cdots,\mathbf{a}(\theta_Q)]$ with $\mathbf{a}(\theta_i)$ denoting the array response vector of the ith ray; $\mathbf{B}(n) = diag(\beta_1(n),\cdots,\beta_Q(n))$ with $\beta_i(n)$ being the complex fading amplitudes of the ith ray during the nth burst; $\tilde{\mathbf{G}}(\tau) = [\tilde{\mathbf{g}}(\tau_1),\cdots,\tilde{\mathbf{g}}(\tau_Q)]$ with $\tilde{\mathbf{g}}(\tau_i) = \mathbf{S}_t^T \cdot g(\tau_i)$ being the convolution between the training sequence matrix $\mathbf{S}_t^T$ and the pulse-shaping function $g(\tau_i)$. $N^{(n)}$ is oversampling matrix of noise.

In the application of wireless communication, the Q fading rays are usually assumed to be mutually uncorrelated, and their fading amplitudes are assumed to be zero-mean complex Gaussian distribution [3]. Hence, the covariance matrix of the fading vector $\mathbf{B}(n) = [\beta_1(n),\cdots,\beta_Q(n)]^T$ is

$$E\{\mathbf{B}(n)(\mathbf{B}(n))^H\} = diag(\sigma_1^2,\cdots,\sigma_Q^2) \triangleq \mathbf{Z}$$
$$E\{\mathbf{B}(n)(\mathbf{B}(n))^T\} = 0$$

where σ_i^2 is the average signal power of ray i. $(\cdot)^H$ denotes the Hermitian transpose, $E(\cdot)$ and the overbar $\overline{(\cdot)}$ is used interchangeably to denote the statistical average operation.

We may express the temporal covariance matrix $\mathbf{R}^t$ and the spatial covariance matrix $\mathbf{R}^s$ as [5]

$$\mathbf{R}^t = E\{(\mathbf{X}^{(n)})^T(\mathbf{X}^{(n)})^*\} = \tilde{\mathbf{G}}\mathbf{Z}\tilde{\mathbf{G}}^H + \sigma_n^2\mathbf{I}$$
$$\mathbf{R}^s = E\{(\mathbf{X}^{(n)})(\mathbf{X}^{(n)})^H\} = \mathbf{A}\mathbf{Z}\mathbf{A}^H + \sigma_n^2\mathbf{I}$$

where $(\cdot)^*$ denotes the Hermitian transpose operation, and $\mathbf{A}$ instead of $\mathbf{A}(\theta)$ is used to simplify notation.

3. ALGORITHM FORMULATION

In this section, we define a space-time matrix which includes the information of DOAs and delays of multipath signals.

If we carry out an N_t-point discrete Fourier transform (DFT) on the rows of (2), the resulting frequency domain representation of $\mathbf{X}^{(n)}$ is

$$\mathbf{X}_f^{(n)} = \mathbf{A}(\theta)\mathbf{B}(n)\mathbf{V}^T(\tau)\cdot diag(\tilde{\mathbf{g}}) + \mathbf{N}_f^{(n)} \qquad (3)$$

where $\mathbf{V}(\tau) = [\mathbf{v}(\tau_1),\cdots,\mathbf{v}(\tau_Q)]$ in which $\mathbf{v}(\tau_k) = [1,\psi_k,\cdots,\psi_k^{N_t-1}]^T$ with $\psi_k = exp\{-j\frac{2\pi\tau_k}{N_t}\}$; $\tilde{\mathbf{g}} = [g_0(0),\cdots,g_0(N_t-1)]^T$ with $g_0(k)$ denoting the $(k+1)$th element of DFT of $\mathbf{S}_t^T\cdot\tilde{\mathbf{g}}$, $\mathbf{N}_f^{(n)}$ denotes the DFT of $\mathbf{N}^{(n)}$.

If we post-multiply (3) by $diag(\tilde{\mathbf{g}})^{-1}$, which is referred to as the deconvolution process, (3) becomes

$$\mathrm{H}^{(n)} = \mathbf{X}_f^{(n)}\cdot diag(\tilde{\mathbf{g}})^{-1} = \mathbf{A}(\theta)\mathrm{B}(n)\mathbf{V}^T(\tau) + \mathbf{W}(n) \qquad (4)$$

where $\mathbf{W}(n) = \mathbf{N}_f^{(n)}\cdot diag(\tilde{\mathbf{g}})^{-1}$.

Assume that the number of incident signals Q is known. Let there be $Q < M-1$ multipath signals incident onto a ULA with M sensors. Also assume that the sensors are equally spaced by half-wavelength and thus the array vector $\mathbf{a}(\theta_i) = [1,\phi_i,\cdots,\phi_i^{M-1}]^T$, where $\phi_i = exp\{-j\pi sin\theta_i\}$.

From (4), we construct two submatrices $\mathbf{Y}_1$ and $\mathbf{Y}_2$, which are composed of the first and last $M-1$ rows of matrix $\mathbf{H}^{(n)}$, respectively, namely,

$$\mathbf{Y}_1 = \mathbf{J}_1\mathbf{H}^{(n)} = \mathbf{A}_1\mathbf{B}\mathbf{V}^T + \mathbf{W}_1$$
$$\mathbf{Y}_2 = \mathbf{J}_2\mathbf{H}^{(n)} = \mathbf{A}_1\mathbf{\Theta}\mathbf{B}\mathbf{V}^\mathrm{T} + \mathbf{W}_2$$

where $\mathbf{A_1} = \mathbf{J_1 A}$, $\mathbf{W_1} = \mathbf{J_1 W}$, $\mathbf{J_1} = [\mathbf{I_{M-1}} \quad 0]$, $\mathbf{\Theta} = \mathbf{diag}\{\phi_1, \cdots, \phi_Q\}$ with $\phi_i = exp\{-j\pi sin\theta_i\}$ $(i = 1, \cdots, Q)$. $\mathbf{W_2} = \mathbf{J_2 W}$, $\mathbf{J_2} = [0 \quad \mathbf{I_{M-1}}]$, $\mathbf{I_{M-1}}$ is the $(M-1)$-dimension identity matrix. For simplifying notation, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{V}$ and $\mathbf{W}$ replace $\mathbf{A}(\theta)$, $\mathbf{B(n)}$, $\mathbf{V}(\tau)$ and $\mathbf{W(n)}$, respectively.

Let

$$\begin{aligned} \mathbf{R_1} &= E\{\mathbf{Y_1}^T\mathbf{Y_1^*}\} = \mathbf{VZV^H} + \sigma_n^2\mathbf{I} \\ &= \mathbf{R_{11}} + \sigma_n^2\mathbf{I} \end{aligned} \tag{5}$$

and

$$\mathbf{R_2} = \mathbf{E}\{\mathbf{Y_2^T}\mathbf{Y_1^*}\} = \mathbf{V\Theta ZV^H} \tag{6}$$

Defining the space-time matrix as

$$\mathbf{R} \triangleq \mathbf{R_2 R_{11}^-} \tag{7}$$

where the superscript $(\cdot)^-$ denotes matrix pseudo-inverse (Moore-Penrose inverse), then we have following theorem 1.

Theorem 1 Assume that $\mathbf{V}$ is full rank matrix, and there are no same elements on the diagonal of matrix $\mathbf{\Theta}$. Then the Q nonzero eigenvalues of $\mathbf{R}$ are equal to the Q diagonal elements of matrix $\mathbf{\Theta}$, and the corresponding eigenvectors are equal to the corresponding column vectors of $\mathbf{V}$, namely, $\mathbf{RV} = \mathbf{V\Theta}$.

Proof: Because $\mathbf{\Theta}$, and $\mathbf{V}$ are full rank matrices, it easy to know $rank(\mathbf{R_{11}})= Q$.

Let us define

$$\mathbf{U} = \mathbf{V}$$

and

$$\mathbf{C} = \mathbf{ZV^H}.$$

Since $rank(\mathbf{U}) = Q$ and $rank(\mathbf{C})= Q$, it can be easily shown that the Moore-Penrose inverse $\mathbf{R_{11}^-}$ is given by

$$\begin{aligned} \mathbf{R_{11}^-} &= \mathbf{C^H(CC^H)^{-1}(U^H U)^{-1}U^H} \\ &= \mathbf{VZ(ZV^H VZ)^{-1}(V^H V)^{-1}V^H} \end{aligned} \tag{8}$$

where the superscript $(\cdot)^{-1}$ denotes matrix inverse.

From (6)-(8), we have the following equation

$$\begin{aligned} \mathbf{RV} &= \mathbf{R_2 R_{11}^- V} \\ &= \mathbf{(V\Theta ZV^H)(VZ(ZV^H VZ)^{-1}(V^H V)^{-1}V^H)V} \\ &= \mathbf{V\Theta(ZV^H VZ)(ZV^H VZ)^{-1}(V^H V)^{-1}(V^H V)} \\ &= \mathbf{V\Theta} \end{aligned} \tag{9}$$

Remarks:

From theorem 1, we can make several immediate conclusions, as follows:

1) The elements on **Theta**'s diagonal are the nonzero eigenvalues of $\mathbf{R}$, and the columns of $\mathbf{V}$ are the eigenvectors of $\mathbf{R}$. Therefore, the DOAs and delays can be estimated by the eigenvalues and eigenvectors of the space-time matrix $\mathbf{R}$, respectively.

2) The paring of the estimated DOAs and delays is automatically determined by the relationship of eigenvalue and eigenvector.

These are the key relationship in the development of the space-time matrix method and its properties.

The space-time matrix method can be summarized as follows.

1) Get data matrixes $\hat{\mathbf{Y}}_1$ and $\hat{\mathbf{Y}}_2$.

2) Calculate $\hat{\mathbf{R}}_1$ and $\hat{\mathbf{R}}_2$ by $\hat{\mathbf{Y}}_1$ and $\hat{\mathbf{Y}}_2$.

3) Compute the eigendecomposition of $\hat{\mathbf{R}}_1$

$$\hat{\mathbf{R}}_1\hat{\mathbf{E}} = \hat{\mathbf{E}}\mathbf{\Lambda}$$

where $\mathbf{\Lambda} = \mathbf{diag}\{\lambda_1, \cdots, \lambda_{\mathrm{N_t}}\}$, $\lambda_1 \geq \cdots \geq \lambda_Q > \lambda_{Q+1} = \cdots = \lambda_{N_t} = \sigma_n^2$, and the columns of $\hat{\mathbf{E}}$ are the eigenvectors of $\hat{\mathbf{R}}_1$.

4) Estimate $\mathbf{R_{11}}$, denoting $\hat{\mathbf{R}}_{11}$, by $\hat{\mathbf{R}}_{11} = \hat{\mathbf{R}}_1 - \sigma_n^2\mathbf{I}$.

5) Compute the Moore-Penrose inverse of $\hat{\mathbf{R}}_{11}$, denoting $\hat{\mathbf{R}}_{11}^-$, and the space-time matrix $\hat{\mathbf{R}} = \hat{\mathbf{R}}_2\hat{\mathbf{R}}_{11}^-$.

6) Solve for the eigensystem; $\hat{\mathbf{R}}\hat{\mathbf{E}} = \hat{\mathbf{E}}\mathbf{\Gamma}$, where $\mathbf{\Gamma} = \mathbf{diag}\{\nu_1, \cdots \nu_{\mathrm{Q}}\}$ with ν_i is the nonzero eigenvalue of $\hat{\mathbf{R}}$, and the columns of $\hat{\mathbf{E}}$ are the eigenvectors of $\hat{\mathbf{R}}$.

7) Estimate the DOAs θ_k by making use of the kth eigenvalue of $\hat{\mathbf{R}}$. Use the second element of the kth eigenvector of $\hat{\mathbf{R}}$ to estimate the delay τ_k.

4. PERFORMANCE ANALYSIS

In this section, we investigate the performance of the proposed method for joint DOA and delay estimate. We first briefly discuss the mean square error (MSE) on the DOA and delay estimates. Finally, the computational complexity of the proposed algorithm is addressed as well.

4.1 Mean Square Error in the Joint DOA and Delay estimate

4.11 Mean square error in the delay estimate

Now, we characterize the error in the delay's as a result of using an estimated matrix.

The proposed method utilizes the eigendecomposition of the matrix $\mathbf{R}$,

$$\mathbf{R} = \mathbf{E\Lambda E^H}$$

where $\mathbf{E} = [\mathbf{s_1}, \cdots, \mathbf{s_Q}]$ with $\mathbf{s_i}$ being the orthonormal eigenvector of the matrix $\mathbf{R}$, $\mathbf{\Lambda} = \mathbf{diag}\{\lambda_1, \cdots, \lambda_{\mathrm{Q}}\}$ with λ_i being he eigenvalue of the matrix $\mathbf{R}$. In this paper, we analyze the effect of using an estimated matrix obtained by the space-time matrix method,

$$\hat{\mathbf{R}} = \hat{\mathbf{E}}\hat{\mathbf{\Lambda}}\hat{\mathbf{E}}^{\mathrm{H}}$$

where $\hat{\mathbf{E}} = [\hat{\mathbf{s}}_1, \cdots, \hat{\mathbf{s}}_{\mathrm{Q}}]$, $\hat{\mathbf{\Lambda}} = diag\{\hat{\lambda}_1, \cdots, \hat{\lambda}_Q\}$.

Let $\hat{\mathbf{s}}_k = \mathbf{s_k} + \mathbf{\Delta s_k}$, and $\hat{\lambda}_k = \lambda_k + \Delta\lambda_k$. The analysis makes use of the asymptotic properties of the errors $\Delta\mathbf{s_k}$. According to [11, 12], we have the following equations:

$$E\{\Delta \mathbf{s_k} \mathbf{\Delta s_j^H}\} \approx \frac{\lambda_k}{N_t} \sum_{m=1, m\neq k}^{Q} \frac{\lambda_m}{(\lambda_k-\lambda_m)^2} \mathbf{s_m s_m^H} \delta_{\mathbf{kj}}$$

$$E\{\Delta \mathbf{s_k} \mathbf{\Delta s_j^T}\} \approx -\frac{\lambda_k \lambda_j}{N_t(\lambda_k-\lambda_j)^2} \mathbf{s_j s_k^T}(\mathbf{1}-\delta_{\mathbf{kj}})$$

where δ_{kj} is the Kronecker delta, and the terms neglected in the approximations are $\emptyset(\frac{1}{N_t})$.

An error $\Delta\psi_k$ in the eigenvector $\mathbf{s_k}$ of the matrix $\mathbf{R}$ due to errors in the matrix $\mathbf{R}$ can be written as

$$\Delta\psi_k = \mathbf{e_2 \Delta s_k}$$

where $\mathbf{e_2} = [\mathbf{0}, \mathbf{1}, \mathbf{0}, \cdots, \mathbf{0}]$.

The mean square error is given by

$$\begin{aligned} E\{|\Delta\psi_k|^2\} &= E\{(\mathbf{e_2 \Delta s_k})(\mathbf{e_2 \Delta s_k})^{\mathrm{H}}\} \\ &= \frac{\lambda_k}{N_t} \sum_{m=1, m\neq k}^{Q} \frac{\lambda_m}{(\lambda_k-\lambda_m)^2} \mathbf{e_2 s_m s_m^H e_2^T} \end{aligned}$$

The quantity of interest is τ_k. It can be shown that the error in the delay τ_k is related to that the error in ψ_k by the expression given below [11]

$$\overline{|\Delta\tau_k|^2} = (\frac{N_t}{2\pi})^2 \frac{\overline{|\Delta\psi_k|^2} - Re\{(\psi_k^*)^2 \overline{|\Delta\psi_k|^2}\}}{2} \quad (10)$$

To obtain the mean squared error in the delay estimate, an expression for $(\psi_k^*)^2\overline{|\Delta\psi_k|^2}$ is required,

$$(\psi_k^*)^2\overline{|\Delta\psi_k|^2} = \frac{\lambda_k}{N_t} \sum_{m=1, m\neq k}^{Q} \frac{\lambda_m}{(\lambda_k-\lambda_m)^2} S_{km}$$

where $S_{km} = \mathbf{e_2}[(\mathbf{s_k^* s_k^H}) \odot (\mathbf{s_m s_m^H})]\mathbf{e_2^T}$, $\mathbf{A} \odot \mathbf{B}$ denotes the Hadamard Product of $\mathbf{A}$ and $\mathbf{B}$.

4.12 Mean square error in the DOA estimate

According to the discussion of the mean square error in the delay estimate, we can easily obtain the mean square error in the DOA estimate as follows:

$$\overline{|\Delta\theta_k|^2} = (\frac{\lambda}{2\pi d \cos\theta_k})^2 \frac{\overline{|\Delta\lambda_k|^2} - Re\{(\lambda_k^*)^2 \overline{|\Delta\lambda_k|^2}\}}{2} \quad (11)$$

where

$$\overline{|\Delta\lambda_k|^2} = \mathbf{r_k^H}(\textstyle\sum_{\mathbf{i=1}}^{\mathrm{M}} \sum_{\mathbf{j=1, j\neq i}}^{\mathrm{M}} \mathbf{x_{ki} x_{kj}} \mathrm{D_1})\mathbf{r_k^*}$$

$$\mathbf{D_1} = (\mathbf{W_2} - \lambda_{\mathbf{k}} \mathbf{W_1})\overline{\mathbf{\Delta s_i s_J^T}}(\mathbf{W_2} - \lambda_{\mathbf{k}} \mathbf{W_1})^{\mathrm{T}}$$

$$(\lambda_k^*)^2\overline{|\Delta\lambda_k|^2} = \mathbf{r_k^H}(\sum_{\mathrm{i=1}}^{\mathrm{M}} \sum_{\mathrm{j=1, j\neq i}}^{\mathrm{M}} \mathbf{x_{ki} x_{kj} D_2})\mathbf{r_k^*}$$

$$\mathbf{D_2} = (\mathbf{W_2} - \lambda_{\mathbf{k}}^* \mathbf{W_1})\overline{\mathbf{\Delta s_i s_J^T}}(\mathbf{W_2} - \lambda_{\mathbf{k}}^* \mathbf{W_1})^{\mathrm{T}}$$

$$\mathbf{W_1} = [\mathbf{I_{N_t-1}} \quad 0] \quad \mathbf{W_2} = [0 \quad \mathbf{I_{N_t-1}}]$$

$\mathbf{x_k}$ is the eigenvector of $\mathbf{E_1^- E_2} \triangleq \mathbf{P}$ ($\mathbf{E_1} = \mathbf{W_1 E}, \mathbf{E_2} = \mathbf{W_2 E}$) corresponding the eigenvalue λ_k and $\mathbf{q_k}$ is the left eigenvector, i.e., $\mathbf{P x_k} = \lambda_{\mathbf{k}} \mathbf{x_k}$ and $\mathbf{q_k P} = \lambda_{\mathbf{k}} \mathbf{q_k}$. In addition, $\mathbf{q_k}$ and $\mathbf{x_k}$ satisfy $\mathbf{q_k x_k} = \mathbf{1}$.

4.2 Computational Complexity

Table 1: *Comparisons of the Computational Complexity of Several Algorithms*

Algorithms	Computational complexity
JADE-MUSIC	$\emptyset(M^3N_t^3) + \emptyset(M^2N_t^2 g_t g_s)$
JADE-ESPRIT	$\emptyset(M^3N_t^3)$
TST-MUSIC	$\emptyset(N_t^3) + \emptyset(N_t^2 g_t)$
TST-ESPRIT	$\emptyset(N_t^3) + \emptyset(N_t^2)$
the proposed method	$\emptyset(N_t^3)$

We use $\emptyset(M^3)$ to represent the order of M^3. The major computational complexity of the proposed method includes the EVD of $\hat{\mathbf{R}}_1$, $\hat{\mathbf{R}}$ of orders $\emptyset(N_t^3)$. Table 1 presents the computational complexity of the previous algorithms and the proposed space-time matrix method, respectively, including the eigenvalue decomposition. g_t and g_s are the number of searches conducted along the delay axis and the DOA axis, N_t is the length of the extracted training sequence, M is the number of isotropic sensors, Q is total number of rays present in the communication system.

5. SIMULATION RESULTS

In this section, we conduct some simulations to validate the proposed space-time matrix method. Assume the narrowband signals which are transmitted through three rays ($Q = 3$) and received by a ULA with six elements $M = 6$. The sensor displacement d is taken to be half the wavelength of the signal waves. Assuming the GSM system model, the GMSK modulated signals are tested. The received signal $\mathbf{x(t)}$ is sampled during 20 data bursts. Let the DOAs and the propagation delays be $[-25°, 8°, 27°]$, and $[0.19, 1.01, 0.56]T_s$, respectively, where $T_s = 3.68\mu s$ is the symbol period of the GSM system. The oversampling factor $P = 4$, and the training sequence of each burst was truncated from the sixth training bit to the 21th training bit to keep the samples of the training sequence from being corrupted by the data bits. The average fading amplitudes of the three rays are equal and normalized to 0dB with randomly selected but constant fading phases. The average power of the additive Gaussian noise is adjusted to achieve the required SNR.

We use root mean square error (RMSE), which is defined as $\sqrt{E\{(\hat{\theta}_k - \theta_k)^2\}}$ and $\sqrt{E\{(\hat{\tau}_k - \tau_k)^2\}}$($k = 1, 2, 3$), as the performance measure.

Figure 1 and 2 illustrate the scattergrams of the space-time matrix method, and the JADE-ESPRIT algorithm, respectively, based on 200 independent trials with the signal-to-noise ratio (SNR) equal to 0dB. We can observe from these figures that the space-time method can provide a more precise DOA-delay estimate than JADE-ESPRIT algorithm.

Figure 3 and 4 give the RMSE curves of the aforementioned algorithms for DOA and delay estimation of source 1, respectively, with SNRs ranging from 0dB to 25dB. In these figures, Cramer-Rao Bound (CRB) and theoretical curves (which are computed according to (10) and (11)) are also shown. As shown in Fig.3-4, the proposed algorithm can provide better performance such as smaller estimation error and superior noise-resistant capability for joint DOA and delay estimation.

6. CONCLUSIONS

In this paper, a space-time matrix which includes the information of DOAs and delays of multipath signals in wireless communication environments is defined. Making use of the proposed method, DOAs and delays can be estimated by eigenvalues and eigenvectors of the defined space-time matrix. The paring of the estimated DOAs and delays is automatically determined by the relationship between eigenvalues and eigenvectors. The space-time matrix method can avoid vectorization (column stacking), Khatri-Rao product, Kronecker product, which are complicated mathematic operation. Compared with the previous works, the proposed method has a lower computational complexity. Simulation results (Fig.1-4) show the proposed method has superior performance, such as smaller estimation error and better robustness to SNR change etc.

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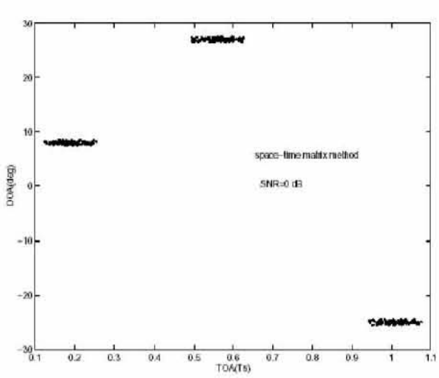

Fig.1: *The Performance of the Space-Time Method.*

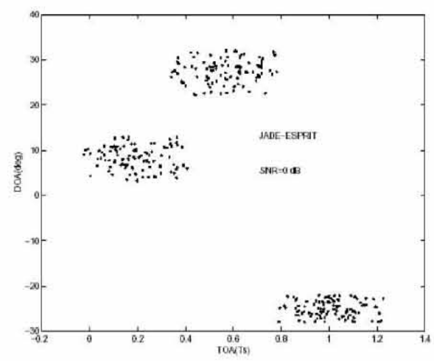

Fig.2: *The Performance of the JADE-ESPRIT.*

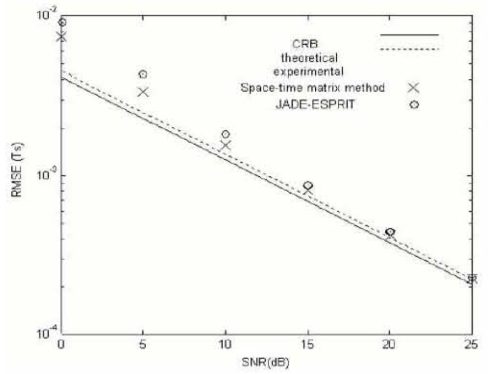

Fig.3: *The Performance for the Delay estimation of Source 1 in Simulation Example.*

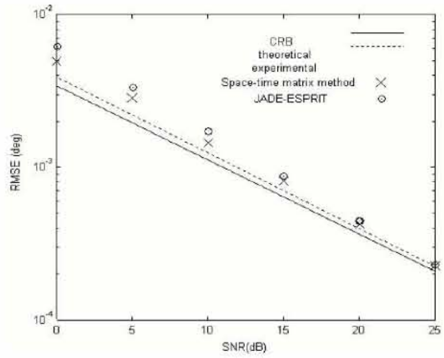

Fig.4: *The Performance for the DOA estimation of Source 1 in Simulation Example.*

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