Robust Digital Control for Boost DC-DC Converter

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ABSTRACT

If the duty ratio, load resistance and input voltage in boost DC-DC converter are changed, the dynamic characteristics is varied greatly, that is, boost DC-DC converter has non-linear characteristics. In many applications of DC-DC converters, the load cannot be specified in advance, and it will be changed suddenly from no load to full load. In the boost DC-DC converter system used a conventional single controller cannot be adapted to change dynamics and it occurs large output voltage variation. In this paper, an approximate 2-Degree-of-Freedom (2DOF) digital controller for suppressing the change of step response characteristics and variation of output voltage in the load sudden changes is proposed. Experimental studies using micro-processor for controller demonstrate that this type of digital controller is effective to suppress variations.

Keywords: Boost DC-DC Converter, Approximate 2DOF, Robust Digital Control, Micro-Processor

1. INTRODUCTION

In many applications of DC-DC converters, load cannot be specified in advance, i.e., their amplitudes are suddenly changed from the zero to the maximum rating. In a boost DC-DC converter, if the duty ratio, load resistance and input voltage are changed, the dynamic characteristics are varied greatly, that is, the boost DC-DC converter has non-linear characteristics. Usually, the controller of the boost DC-DC converter is designed to the approximated linear controlled object at one operating point. In a non-linear boost DC-DC converter system, it is not enough for the design of controller considering only one operating point. As a technique to improve dynamic performance, the gain-scheduled control is proposed. This method switch many controllers designed to many operating points. However, it requires complicated control routine when controllers are implemented to micro-processor. Then, the controller which can cover sudden load changes and dynamic characteristics changes with only one controller is needed. PI control etc.[1-6] are conventionally used for the boost DC-DC converter, but robustness is not enough. In order to improve robustness, various methods[7-11] are proposed, but these have not improved so much, either.

In this paper, an approximate 2-degree-of-freedom (2DOF) design method robust enough which suppresses very small the output voltage regulation at load sudden change is proposed. Robust control method using an approximate 2DOF for improving start-up characteristics and load sudden change characteristics of DC-DC converters has been proposed [12-13]. However, it was applied to buck DC-DC converter. The Boost DC-DC converter is non-linear system and the dynamic characteristics are changed at each operating point. The design method of the approximate 2DOF digital controller which considering much operating points with one controller is proposed. By this method, even if the dynamic characteristics of converter are change, the variation of the output voltage and dynamic characteristic can be suppressed enough. This controller is actually implemented on a micro-processor and is connected to the boost DC-DC converter. Experimental studies demonstrate that the digital controller designed by proposed method satisfies the given specifications and is useful.

2. BOOST DC-DC CONVERTER

2.1 State-space model of boost DC-DC converter

The boost DC-DC converter is shown in Fig. 1. In Fig.1, \( v_i \) is input AC voltage, \( C_{in} \) is smoothing capacitor, \( L_0 \) is main switch, \( L_0 \) and \( D_0 \) are boost inductance and diode, \( C_0 \) is output capacitor, \( R_L \) is output load resistance, \( i_L \) is inductor current, \( i_o \) is output current, and \( v_o \) is output voltage. Here \( C_{in} \) is 1[\mu F], \( L_0 \) is 150[\mu H] and \( C_0 \) is 940[\mu F]. Using averaging method, the state equation of the controlled object in Fig.1 becomes as follows [14]:

\[
\frac{d}{dt}\begin{bmatrix}
 v_o \\
 i_L
\end{bmatrix} = \begin{bmatrix}
 -\frac{1}{R_L C_0} & \frac{1-\mu}{L_0 C_0} \\
 \frac{1-\mu}{L_0} & \frac{R_L}{L_0}
\end{bmatrix}\begin{bmatrix}
 v_o \\
 i_L
\end{bmatrix} + \begin{bmatrix}
 0 \\
 \frac{V_o}{L_0}
\end{bmatrix}
\]

\[ + \begin{bmatrix}
 v_o \\
 0
\end{bmatrix} \begin{bmatrix}
 0 \\
 \frac{1}{L_0}
\end{bmatrix} + i_L \begin{bmatrix}
 -\frac{1}{C_0} \\
 0
\end{bmatrix} \]  

(1)
Here equivalent resistance of inductor $R_0$ is 1.8[Ω] and rectified input voltage $V_i$ is 141[VDC]. $\mu$ is duty ratio. $\mu = \mu_s + \mu_q$ is the value at each operating point and $\bar{\mu}$ is the small variation at operating points. The boost DC-DC converter has non-linear characteristics because this equation has the product of state variable and duty ratio.

**Fig.1: Boost DC-DC Converter**

### 2.2 Static characteristics of boost converter

In the static characteristics, the differential values of state variables and $\bar{\mu}$ are zero. Then average of output voltage $V_s$ and inductor current $I_s$ at operating points becomes as follows:

$$V_s = \frac{1}{1 - \frac{1}{R_0} - \frac{1}{(1 - \mu_s)^2 R_L}} V_i$$

$$I_s = \frac{1}{R_L} \frac{V_s}{1 - \mu_s}$$

Eq. (2) turns out that the boost DC-DC converter is non-linear system. The static characteristics of the boost DC-DC converter is changed greatly with load resistances, and it influences the dynamic characteristics of converter. In addition, the static characteristics will be changed with input voltage variations.

$$x(t) = A_c x(t) + B_c u(t)$$

$$y(t) = C_s x(t)$$

Where

$$A_c = \begin{bmatrix} -\frac{R_0}{L_0} & -\frac{1}{L_0 C_0} \\ \frac{1}{L_0 C_0} & \frac{1}{L_0 C_0} \end{bmatrix}, B_c = \begin{bmatrix} V_o \\ -\frac{V_v}{L_0} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} i_z(t) \\ v_s(t) \end{bmatrix}, u(t) = \bar{\mu}(t), C_c = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

From this equation, each matrix of the boost converter depends on duty ratio $\mu_s$. Therefore, the initial value and converter response at the operating point will be changed depending on duty ratio and operating point variations. The load changes of the controlled object and output voltage change are considered as parameter changes in eq. (3). Such parameter changes can be transformed to equivalent disturbances $q_u$ and $q_v$ as shown in Fig.5. Therefore, what is necessary is just to constitute the control systems whose pulse transfer functions from equivalent disturbances $q_u$ and $q_v$ to the output $y$ become as small as possible in their amplitudes, in order to robustize or suppress the influence of these parameter changes.

The controller of the DC-DC boost converter used as inverter power supplies for EV (electric vehicle) and Air Conditioner, etc. is designed. Then the controller which satisfies the following specifications will be designed.

1. Input voltage $v_i$ is 100[VAC], And output voltage $v_o$ changes from 240[VDC] to 385[VDC].
2. Step responses are almost the same at resistive loads where $300 \leq R_L \leq 5k$ [Ω], And an over-shoot is less than 10[%] in the step response.
3. The dynamic load response is smaller than 10[VDC] against change of load between 30~500[W].

Under these specifications, three operating points for deciding the controller was determined as follows:

**Point 1:** Output voltage is 385[VDC], Resistive load is 5[kΩ]  
**Point 2:** Output voltage is 385[VDC], Resistive load is 300[Ω]  
**Point 3:** Output voltage is 240[VDC], Resistive load is 300[Ω]

The gains and phases characteristics of the boost DC-DC converter are different at each operating point. One approximate 2DOF controller is designed by considering these operating points.

### 3. DIGITAL ROBUST CONTROLLER

#### 3.1 Discretion of controlled object

The continuous system of eq. (3) is transformed into the discrete system as follows:

$$x_d(k + 1) = A_d x_d(k) + B_d u(k) + q_u(k)$$

$$y(k) = C_d x_d(k) + q_v(k)$$

Where

$$A_d = [e^{A_c T}] , B_d = \int_0^T e^{A_c \tau} B_c d\tau , C_d = C_c$$

And sampling period $T_s = 10[\mu s]$. Here, in order to compensate the delay $L_d = 0.997T_s$ by ADC conversion time and micro-processor operation time etc., one delay (state $\xi_1$) is introduced to input of the controlled object. And, more one delay (state $\xi_2$) is also introduced to input of the controlled object for the
current feedback equivalent conversion. Then, a new controlled object with two delays is shown in Fig.2. The state-space equation is described as follows:

\[
x_{dw}(k+1) = A_{dw}x_{dw}(k) + B_{dw}v(k) \\
y(k) = C_{dw}x_{dw}(k)
\]

Where

\[
A_{dw} = \begin{bmatrix} A_w & B_w \\ 0 & 0 \end{bmatrix}, B_{dw} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

\[
C_{dw} = \begin{bmatrix} C_c & 0 \\ 0 & 0 \end{bmatrix}, \xi_2(k-1) = \xi_1(k)
\]

\[
A_w = e^{A_w(T-L_d)} \int_0^{L_d} e^{A_c \tau} B_c d\tau \\
= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \\
0 & 0 & 0
\]

\[
B_w = \begin{bmatrix} \frac{T-L_d}{1} e^{A_c \tau} B_c d\tau \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} \\ b_{21} \end{bmatrix} \\
1
\]

\[
x_{dw}(k) = \begin{bmatrix} x_d(k) \\ \xi_1(k) \\ \xi_2(k) \end{bmatrix} = \begin{bmatrix} v_o(k) \\ \nu_L(k) \\ \nu(k-1) \end{bmatrix}
\]

Here \( n_1 \) and \( n_2 \) are the zeros of the discrete-time controlled object. It shall be specified that the relation of \( H_1 \) and \( H_2, H_3 \) become \([H_1] \gg [H_2, [H_3]]\). Then \( W_{ry} \) can be approximated to the following first-order discrete model:

\[
W_{ry}(z) \approx W_m(z) = \frac{1 + H_1}{z + H_1}
\]

This target characteristic \( W_m \) is realizable by applying a state feedback shown in Fig.3:

\[
v = -Fx_{dw} - G(z - H_4)r
\]

Where

\[
F = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix} \\
G = \frac{1}{C_{dw}I - (A_{dw} - B_{dw}F)B_{dw}} \frac{1}{1 + H_4}
\]

The current feedback is used in Fig.3. This is transformed into voltage and control input feedbacks without changing the pulse transfer function \( W_{ry} \) by an equivalent transform using following equation:

\[
-f_2x_2(k) = -\frac{f_2}{a_{12}}(x_1(k + 1) - a_{12}x_1(k) - a_{13}\xi_1(k) - b_{12}\xi_2(k))
\]

Using this equation, Fig.3 is transformed to Fig.4, and then to Fig.5.

3.2 Design method for approximate 2DOF digital controller

To the new controlled object of eq. (5), the model matching control system is constituted using the state feedback as shown in Fig.3.

The system of Fig.3, transfer function from reference input \( r \) to output \( y \) is described as follows:

\[
W_{ry}(z) = \frac{(1 + H_1)(1 + H_2)(1 + H_3)}{(z + H_1)(z + H_2)(z + H_3)} \times \frac{(z - n_1)(z - n_2)(1 + H_4)}{(1 - n_1)(1 - n_2)(z + H_4)}
\]
The system in Fig.5 is constituted as shown in Fig.6. The system added the inverse system and the filter to method for constituting the model matching system. The system reconstituted with inverse system and filter is defined as follows:

\[
ff_1 = -f_1 + \frac{f_2}{a_{12}} \left( a_{11} - f_4 + \frac{f_2b_{11}}{a_{12}} \right), \quad ff_2 = -\frac{f_2}{a_{12}}
\]

\[
ff_3 = -f_3 + \frac{f_2a_{13}}{a_{12}}, \quad ff_4 = -f_4 + \frac{f_2b_{11}}{a_{12}}
\]

(10)

Where \( f_i, i = 1, \ldots, 4 \) are state feedback gain \( F = [f_1 \ f_2 \ f_3 \ f_4] \) that obtained from state feedback method for constituting the model matching system. The system added the inverse system and the filter to the system in Fig.5 is constituted as shown in Fig.6.

From eq. (15), (16), it turns out that the characteristics from \( r \) to \( y \) can be specified with \( H_1 \) and the characteristics from \( q_u \) and \( q_y \) to \( y \) can be independently specified with \( k_z \). That is the system in Fig.6 is an approximate 2DOF system, and its sensitivity against disturbances becomes lower with the increase of \( k_z \). If \( k_z \) is sufficiently large, the system becomes robust for parameter changes and uncertainty. Equivalent conversion of the controller in Fig.6, we obtain Fig.7.

Then, substituting a system of Fig.5 to Fig.7, approximate 2DOF digital integral type control system will be obtained as shown in Fig.8.

In Fig.8, the parameters of the controller are as follows:

\[
k_1 = ff_1 - \frac{G(H_4 + ff_4)k_z}{1 + H_2}, \quad k_2 = ff_2 - \frac{Gk_z}{1 + H_2}
\]

\[
k_3 = ff_3, k_4 = ff_4, k_i = G(H_4 + ff_4)k_z
\]

\[
k_{1i} = Gk_2, k_{1r} = G, k_{2r} = G(H_4 + ff_4)
\]

(18)

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Then, substituting a system of Fig.5 to Fig.7, approximate 2DOF digital integral type control system will be obtained as shown in Fig.8.

In Fig.8, the parameters of the controller are as follows:
Next, how to determine $k_z$ using the root locus characteristics will be shown. The root locus of this system is derived from the closed loop characteristics of Fig.8. The root locus characteristics are shown in Fig.9.

In this figure, poles $p_1$, $p_2$, and $p_3$ of this system should be inside of unit circle in z-place. In this situation, if $k_z$ is increase from 0 to 0.5, poles are move along these allows. It shows the $k_z$ should be under 0.3 for system stability. From Fig.9, $k_z$ is set at 0.3. Then the parameters of controller become as follows using eq. (8), (10), (17):

$$
k_1 = 19.89597, k_2 = -36.74662
$$

$$
k_3 = -0.355444, k_4 = 0.1595481
$$

$$
k_i = 0.0001961167, k_{iz} = 0.003293415
$$

Fig.9: Root locus

4. EXPERIMENTAL RESULTS

Experimental setup system is shown in Fig.10. In this experiment, the micro-processor SH7216 by Renesas Electronics Corp. is used. The experiment results at step responses and load sudden change is shown in Fig.11 and Fig.12. In Fig.11, rising time in step responses is about 80[ms], and it turns out that even if the operating point changes, the step responses are not change and it satisfy specification without over-shoot. In Fig.12, the output voltage variation in sudden load change is about 4.5[V] (1.17%). The experimental result at load sudden change used usual PI controller is shown in Fig.13. In this figure, the output voltage variation in sudden load change is over 20[V] (5.19%), and recovery time are over 100[ms]. From these results, the control system using PI controller cannot satisfy specification. As a result, it turns out that proposed one is effective practically.

5. CONCLUSIONS

In this paper, the concept of controller for non-linear boost DC-DC converter to attain good robust-

![Experimental setup system](image)

Fig.10: Experimental setup system

![Experimental results of step responses](image)

Fig.11: Experimental results of step responses

![Experimental results of sudden load change](image)

Fig.12: Experimental results of sudden load change
Robust Digital Control for Boost DC-DC Converter

Fig. 13: Experimental results of sudden load change, where using digital PI controller

ness was given. The proposed digital controller was implemented on the micro-processor. The DC-DC converter built-in this micro-processor was manufactured. It was shown from experiments that the proposed approximate 2DOF digital controller can suppress the variation of step responses in each operating point. And proposed method can reduce output voltage variation in sudden load change better than PI controller. This fact demonstrates the usefulness and practicality of our proposed method. As a result, the proposing method is an innovative method applicable to many boost converters of which capacities differ.

The output voltage regulation at input voltage change is not checked. It is a future subject to check this.

References


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