

Periodically Time-Varying Realizations of Multirate Converters for Hardware Reduction

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ABSTRACT

Realizations of multirate FIR filters are proposed using periodically time-varying (PTV) structures. By exploiting the computational redundancy of the filtering operation in a multirate filter, it is possible to implement the filter with much less hardware. In the proposed implementations, the number of multiply-and-add units is reduced by making several coefficients share one multiplier-and-add unit in a periodic fashion. Specifically, each multiply-and-add unit performs different coefficient scalings at different time instants within a period. Compared to the direct form realization, the proposed realizations reduce the hardware of an interpolator and a decimator by a factor of approximately U and M , respectively, while retaining the same processing speed, where U and M are the upsampling and downsampling factors, respectively. For a rational sampling rate conversion by a factor of U/M , where U and M are relatively prime, the proposed realization offers hardware reduction by a factor of approximately UM , compared to the conventional direct form.

1. INTRODUCTION

Digital multirate converters, which include interpolators and decimators as special cases, perform signal rate conversion equivalent to sampling rate conversion of an analog signal. They have found applications such as in filter banks and transmultiplexers [1]-[2].

Direct-form realizations of interpolator and decimator are inefficient, because some computations are not necessary as they are subsequently discarded prior to the output stage. Several more efficient realizations have been proposed, see the review articles [1] and [3] for examples. The polyphase realization described in [4] uses the same amount of hardware as the direct form but the computation is distributed to different timing phases, so that the computational requirement in each phase is reduced. Consequently,

the processing speed becomes faster. Extension of the polyphase structure to multirate converters was considered in [1][3].

Periodically time-varying (PTV) filters have been studied in various context [5]-[8]. They can be used for realizing interpolators and decimators, as described in [9], which proposed structures based on FIR transposed form with PTV coefficients. They can operate at the same processing speed as the direct form. By arranging several coefficients to periodically share the same hardware multiplier, the number of hardware multipliers is reduced by a factor of U or M , where U and M are the interpolation factor and decimation factor, respectively. However, the method cannot be extended to the case of signal rate conversion of a rational value U/M .

Other realizations using PTV coefficients were described in [10][11], where arrangements were made such that the times originally used for redundant computations are used to help compute the necessary computations. In this fashion, the necessary computations can be distributed over time and space such that each time instant the computation is simplified to the point that no hardware multiplier is needed.

In this paper we propose new realizations for interpolators, decimators, and rational signal rate converters, using fewer filter coefficients which vary periodically. For an interpolator (or decimator) with sampling rate conversion by a factor of U (or M), the number of hardware multipliers is reduced by a factor of approximately U (or M). For a multirate converter with rational conversion rate of U/M , the proposed realizations can reduce the number of hardware multipliers by a factor of approximately UM . The processing speed of the proposed realizations is the same as that of the conventional direct form. Therefore, the proposed structures can serve as hardware-efficient alternative realizations. The synthesis is based on a time-domain approach, by matching the impulse responses of the proposed realizations to those of the target systems.

It should be noted that the proposed realizations have a different objective from that of polyphase realizations. As mentioned above, a polyphase realization reduces the requirement of computational speed without any saving in the hardware, compared to the direct-form realization. Therefore, it is suitable for the case where the speed of the adders/multipliers are not fast enough to keep up with the input/output signals. On the other hand, the proposed realizations

SP2R34: Manuscript received on May 6, 2004 ; revised on October 4, 2004. This work was supported in parts by the National Science Foundation, USA, under grant MIP-8613299 and by the National Electronic and Computer Technology Center (NECTEC), The National Science and Technology Development Agency, Thailand, under grant NT-B-06-4A-13-409.

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assume that the adders/multipliers are faster than necessary, so that they can be shared by several coefficients. Consequently, with proper arrangement of the right coefficient at the right time, the number of adders/multipliers can be significantly reduced.

This paper is organized as follows. Section 2 reviews multirate converters with emphasis on some properties of the impulse responses. This section also discusses some basic FIR filters with PTV coefficients. Section 3 describes the proposed realizations for interpolators and decimators, and shows how the proposed technique can be extended to multirate converter with a rational sampling rate conversion. Section 4 presents some examples and shows how the proposed structures can save hardware, compared to conventional and polyphase realizations. Finally, Section 5 gives some concluding remarks.

2. REVIEW OF MULTIRATE CONVERTERS AND PTV FIR FILTERS

In this section we briefly review multirate converters, which include interpolators and decimators. These are linear PTV systems. In each case, we obtain its impulse responses. We also briefly describe FIR filters with PTV coefficients.

2.1 Interpolator

Figure 1(a) shows a functional block diagram of an interpolator with interpolation factor of U . The input signal $x(n)$ is first upsampled by a factor of U to obtain a signal $w(m)$ with U times higher signal rate. The signal $w(m)$ is then filtered by an interpolation filter with system function $H_I(z)$ to obtain the output $y(m)$. For each input value, the upsampler $\uparrow U$ simply produces an output value equal to the input value followed by $U - 1$ appended zeroes. Specifically,

$$w(m) = \begin{cases} x(m/U), & m = 0, \pm U, \pm 2U, \dots \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Note that we use two different time variables n and m for signals with the slower rate and faster rate, respectively. Note also that we use a thick line and a thin line for signals with the lower rate and the higher rate, respectively. We assume that $H_I(z)$ is an FIR filter with impulse response $h_I(m)$ of length N , so that $H_I(z) = \sum_{i=0}^{N-1} h_I(i)z^{-i}$.

Let $g_I(m, k)$ denote the impulse response of the interpolator due to a unit impulse injected at time k , i.e., $x(n) = \delta(n - k)$ produces an output $y(m) = g_I(m, k)$. The impulse response satisfies

$$g_I(m, k) = g_I(m + U, k + 1) = g_I(m - kU, 0) \quad (2)$$

It can be written in terms of the interpolation filter as

$$g_I(m, 0) = h_I(m) \quad (3)$$

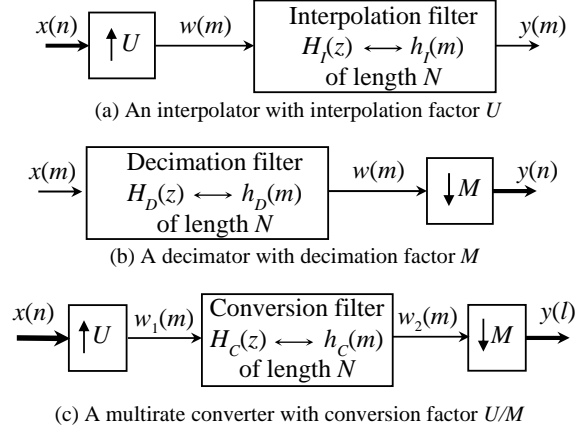


Fig. 1: Functional block diagrams for multirate converters.

2.2 Decimator

Figure 1(b) depicts a functional block diagram of a decimator with decimation factor of M . The input signal $x(m)$ is first filtered by the decimation filter $H_D(z)$ to obtain $w(m)$. We assume that the decimation filter is an FIR filter with impulse response $h_D(m)$ of length N , i.e., $H_D(z) = \sum_{i=0}^{N-1} h_D(i)z^{-i}$. The decimation filter output $w(m)$ is then downsampled by a factor of M to obtain the decimator output $y(n)$. The downsampler simply keeps every M -th value of its input to produce the output, i.e.,

$$y(n) = w(nM) \quad (4)$$

Let $g_D(n, p)$ be the impulse response of the decimator due to an impulse injected at time p , namely, $x(m) = \delta(m - p)$ causes the output $y(n)$ to be $g_D(n, p)$. The impulse response satisfies

$$g_D(n, p) = g_D(n + 1, p + M) \quad (5)$$

A complete characterization of a decimator requires a set of M impulse responses $\{g_D(n, p), p = 0, 1, \dots, M - 1\}$, each of length N/M . These impulse responses are related to the impulse response of the decimation filter by

$$g_D(n, p) = h_D(nM - p) \quad (6)$$

2.3 Rational Multirate Converter

Figure 1(c) is a block diagram for a multirate converter which changes the sampling rate by a rational factor of U/M , where U and M are relatively prime. It consists of an upsampler by a factor of U , followed by a conversion filter $H_C(z) = \sum_{i=0}^{N-1} h_C(i)z^{-i}$ of length N . The output $w_2(m)$ of the conversion filter is then downsampled by a factor of M to obtain the output $y(l)$. Since there are three different signal rates in this system, we use three time variables n , m , and l for the input, the intermediate signals, and the output, respectively.

Let $g_C(l, k)$, $k = 0, 1, \dots, M - 1$, be the impulse response of the multirate converter due to a unit input impulse at time k . It satisfies the periodic condition that

$$g_C(l + U, k + M) = g_C(l, k). \quad (7)$$

It can be verified that this is related to the conversion filter impulse response as

$$g_C(l, k) = h_C(lM - kU) \quad (8)$$

2.4 FIR Filters with PTV Coefficients

Consider an FIR filter with PTV coefficients of period P . Suppose that the length is L , so that there are L PTV coefficients. The filter may be realized using direct form, transposed form [12], or hybrid form [13], as shown in Figure 2, with coefficients $b_i(m)$, $c_i(m)$, or $d_i(m)$, respectively, where $i = 0, 1, \dots, L - 1$. Each PTV coefficient has a period of P , i.e., $b_i(m) = b_i(m + P)$, $c_i(m) = c_i(m + P)$, and $d_i(m) = d_i(m + P)$. The hybrid form is a mixture of the direct and transposed forms. It eradicates the long critical path of the direct form and the required high fan-out of the transposed form. A hybrid form consists of serially connected modules, each of which has a certain number of delay units and coefficients. Figure 2(c) depicts a hybrid form with 2 delays and 2 coefficients per module, as indicated by the dashed box. Note that the PTV FIR filters in Figure 2 are linear PTV systems. Let $h_i(m)$ be the response at time m due to a unit impulse at time i . We can see that $h_{i+P}(m + P) = h_i(m)$, i.e., the response is delayed by P if the unit impulse input is delayed by P . Therefore, it suffices to characterize the filter by P impulse responses $h_i(m)$, $i = 0, 1, \dots, P - 1$.

In the direct form realization, the impulse responses are

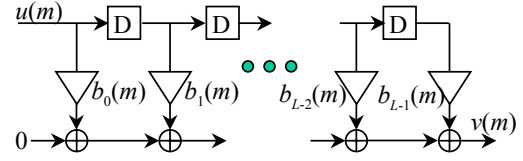
$$\begin{aligned} h_i(m) &= \sum_{j=0}^{L-1} b_j(i + j) \delta(m - (i + j)) \\ &= \begin{cases} b_{m-i}(m), & m = i, \dots, i + (L - 1) \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (9)$$

for $i = 0, 1, \dots, P - 1$. In the case of the transposed form, the impulse responses are

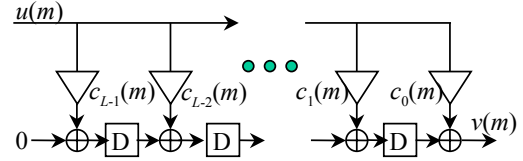
$$\begin{aligned} h_i(m) &= \sum_{j=0}^{L-1} c_j(i) \delta(m - (i + j)) \\ &= \begin{cases} c_{m-i}(i), & m = i, \dots, i + (L - 1) \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (10)$$

for $i = 0, 1, \dots, P - 1$. In the hybrid form in Figure 2(c), let there be Q coefficients per module. For convenience, assume that $L = QR$, where R is an integer. Then, the impulse responses are

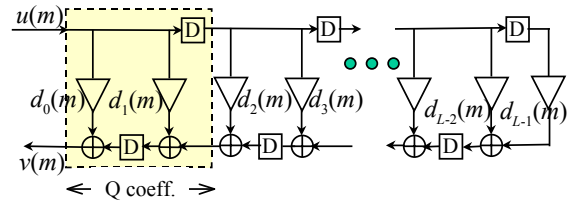
$$h_i(m) = \sum_{j=0}^{L-1} d_j(\lfloor \frac{i+j}{Q} \rfloor + i >_P) \delta(m - (i + j))$$



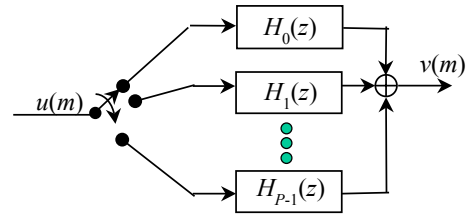
(a) Direct-form PTV FIR filter realization



(b) Transposed-form PTV FIR filter realization



(c) Hybrid-form PTV FIR filter realization with $Q=2$



(d) Equivalent block diagram using commutator structure

Fig. 2: Block diagrams of three FIR filters with PTV coefficients.

$$= \begin{cases} d_{m-i}(\lfloor \frac{m}{Q} \rfloor + i >_P), \\ m = i, \dots, i + (L - 1) \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

for $i = 0, 1, \dots, P - 1$, where $\lfloor t \rfloor$ is the integer part of t and $\langle t \rangle_P$ denotes t modulo P .

We can see that in order to have the same impulse responses, the coefficients in the transposed and hybrid forms are related to those of the direct form by

$$c_j(i) = b_j(i + j), \quad (12)$$

for $i = 0, 1, \dots, P - 1$ and $j = 0, 1, \dots, L - 1$, and

$$d_j(\lfloor \frac{i+j}{Q} \rfloor + i >_P) = b_j(i + j) \quad (13)$$

for $i = 0, 1, \dots, P - 1$ and $j = 0, 1, \dots, L - 1$. The PTV FIR filters above can be modeled by the block diagram in Figure 2(d) which is a commutator structure. It consists of $P - 1$ linear time-invariant (LTI) subsystems. The input values are fed alternately to these subsystems, i.e.,

$$u_i(m) = \begin{cases} u(m), & m = \dots, i - P, i, i + P, i + 2P, \dots \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

The outputs of these subsystems are added to produce the final output $v(m)$. The system function of the i -th LTI subsystem is

$$H_i(z) = \sum_{j=0}^{L-1} h_i(j)z^{-j} \quad (15)$$

where $h_i(m)$ is the impulse response given by (9), (10), or (11).

3. PROPOSED REALIZATIONS

In this section, we propose realizations for the multirate systems in Figure 1. The realizations reduce the number of multipliers, by replacing the filter in each multirate system in Figure 1 with a PTV FIR filter shown in Figure 2 of a shorter filter length, along with some accessory units at the input and/or the output of the system. The equivalence between the original multirate system and the proposed structure is established by equating their impulse responses. We begin with realizations for an interpolator, then obtain realizations for a decimator, and finally realizations for a rational rate converter.

For convenience, we shall call $h_I(m)$, $h_D(m)$, and $h_C(m)$ in Figure 1 the target filters or target impulse responses. As in Section 2, we let the length of the target filters be N .

3.1 Realizations for Interpolators

Consider the target interpolator in Figure 1(a) with interpolation filter $h_I(m)$ of length N . We assume that $N = UL$, where L is an integer and U is the interpolation ratio, such that U and L are relatively prime. Let us partition the target impulse response into U segments, with the i -th segment, $i = 0, 1, \dots, U-1$, being

$$h_I^i(m) = \begin{cases} h_I(m) & m = iL, iL+1, \dots, (i+1)L-1 \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

Based on (16), we observe that $h_I(m)$ can be synthesized by synthesizing U impulse responses $h_I^i(m)$, $i = 0, 1, \dots, U-1$, and appending them together. This can be accomplished by generating U impulses to stimulate U responses. Figure 3(a) shows a realization that can accomplish this, using a comb filter $C_U(z)$ and one FIR filter with L PTV coefficients of period U . Together with the upsampler $\uparrow U$, we obtain a realization for Fig. 1(a). We call this realization PTV Realization for an Interpolator (PTVI). By properly assigning the PTV coefficients of the PTV FIR filter, we can make the realization input/output equivalent to the target interpolator. In the following paragraphs, we derive the impulse response of the PTVI structure and then equate the result to the impulse response of the target interpolator.

In Figure 3(a), the input is first upsampled by a factor of U to obtain $w(m)$, and then passed through

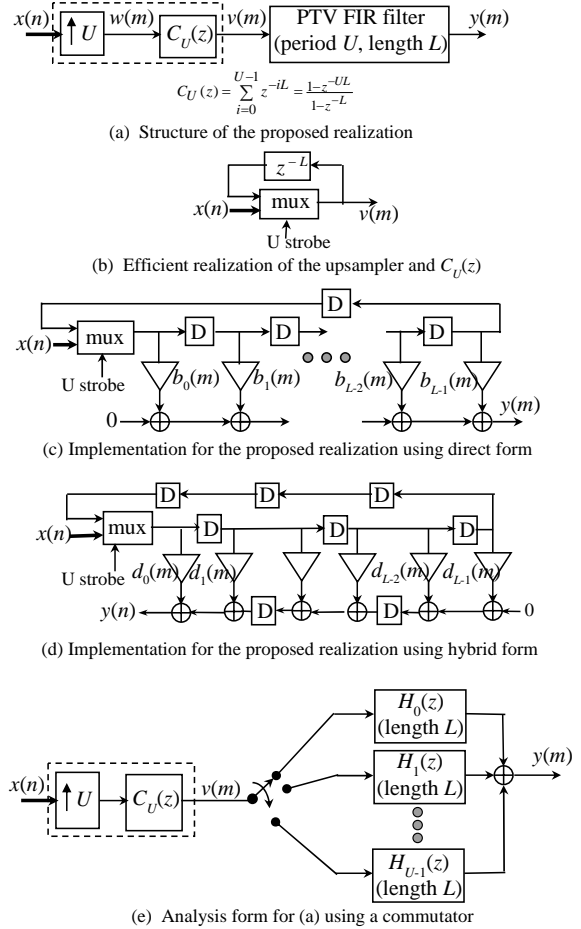


Fig. 3: The Proposed PTV Realization for an Interpolator (PTVI).

a comb filter with system function $C_U(z)$ to obtain $v(m)$ where

$$C_U(z) = 1 + z^{-L} + z^{-2L} + \dots + z^{-(U-1)L}. \quad (17)$$

Consider that an impulse is injected at the input, $x(n) = \delta(n)$. After the upsampler, we have $w(m) = \delta(m)$, which is applied to the comb filter to produce a train of U impulses at instants $m = 0, L, 2L, \dots, (U-1)L$, i.e., $v(m) = \delta(m) + \delta(m-L) + \dots + \delta(m-(U-1)L)$. This train of impulses then excites the PTV FIR filter. For each impulse, the PTV FIR filter responds with a different impulse response of length L . Since the U input impulses entering the PTV FIR filter are separated from each other by exactly L samples, the corresponding U responses will be concatenated together, giving rise to an overall impulse response of length LU . With $LU = N$ and by setting the impulse response due to $\delta(m - iL)$ equal to $h_I^i(m)$ as given by (16), we can obtain the target impulse response $h_I(m)$.

For the above synthesis to work, the impulse $\delta(m - iL)$ for each i must excite each coefficient of the PTV FIR filter with a different value from those for other i , so that the impulse responses will be different for

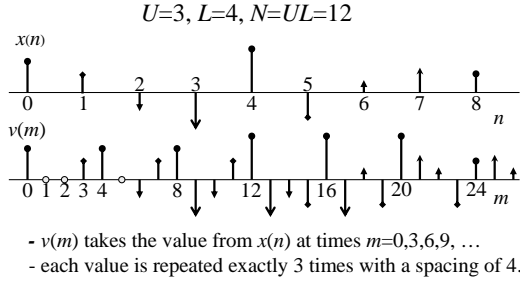


Fig.4: Example of signals for the structure in Fig. 3(b).

different values of i . This implies that $\langle iL \rangle_U$ must be different for different values of i . Such a situation is possible if and only if L and U are relatively prime.

The PTV-FIR filter in Figure 3(a) can be implemented using one of the forms given in Figure 2. The comb filter and the upsampler can also be implemented by a multiplexer (mux) and L delays as shown in Figure 3(b). The multiplexer selects the input from $x(n)$ at times $m = 0, U, 2U, \dots$ and from the feedback path at all other times, using a U-strobe signal. Figure 4 shows an example of the output signal for the structure in Figure 3(b). Here, we let $U = 3$, $L = 4$, and $N = 12$. It is important to point out that although Figure 3(b) has a feedback path, but it works like an FIR system (not IIR system), because for every U -th value the mux selects the input from a new value from $x(n)$, so that the signal in the feedback loop does not propagate forever (it repeats only U times). Although the PTV FIR filter can be realized by any one of the structures in Figure 2, if we use the direct form then the L delay units in the feedback path in Figure 3(b) can share with the $L - 1$ delay units in the direct form. Consequently, only one delay unit is actually needed in the feedback path. The resulting structure is depicted in Figure 3(c). If the hybrid form is used, as shown in Figure 3(d), pipelining is obtained and the processing speed increases. However, more delay units are now needed in the feedback path, compared to that in Figure 3(c).

Let us consider the structure in Figure 3(c). The PTV coefficients $b_l(m)$, $l = 0, 1, \dots, L - 1$ can be determined by comparing the target impulse response and the overall impulse response of Figure 3(c), which can be obtained as follows. Consider an impulse input $x(n) = \delta(n)$. The impulse is processed by the upsampler $\uparrow U$ and the comb filter $C_U(z)$, giving a train of impulses $\sum_{i=0}^{U-1} \delta(m - iL)$. From (9), we know that the PTV FIR filter responds to the input impulse $\delta(m - iL)$ as

$$h_{\langle iL \rangle_U}(m) = \begin{cases} b_{m-iL}(\langle m \rangle_U), \\ m = iL, iL + 1, \dots, (i + 1)L - 1 \\ 0, \quad \text{otherwise} \end{cases} \quad (18)$$

Equating this impulse response to the i -th segment of the target impulse response, we obtain, using (16) and (18),

$$h_I(m) = b_{m-iL}(\langle m \rangle_U) \quad (19)$$

for $m = iL, iL + 1, \dots, (i + 1)L - 1$. With the value of i varying from 0 to $U - 1$, we get all the values of $b_l(m)$ for $l \in [0, L - 1]$ and $m \in [0, U - 1]$ in terms of the target filter impulse response samples $h_I(m)$ using

$$h_I(m) = b_{\langle m \rangle_L}(\langle m \rangle_U) \quad (20)$$

for $m = 0, 1, \dots, N - 1$.

From (20) and using the Chinese Remainder Theorem, we can compute the coefficient $b_l(j)$ in terms of the impulse response as

$$b_l(j) = h_I(\langle lk_1U + jk_2L \rangle_N) \quad (21)$$

for $l = 0, 1, \dots, L - 1$ and $j = 0, 1, \dots, U - 1$, where k_1 and k_2 are the solutions of

$$\langle k_1U \rangle_L = 1 \quad (22)$$

$$\langle k_2L \rangle_U = 1 \quad (23)$$

respectively.

Note that the proposed realizations use only $L = N/U$ multipliers, compared to N multipliers in a direct realization of Figure 1(a) or a polyphase structure. To use the above realizations we require that N is divisible by U and that L and U are relatively prime. Note that these conditions are not at all restrictive. If they are not satisfied, then we can extend the target filter length by appending 0 coefficients until these conditions are satisfied.

Figure 3(e) shows a form, using commutator, of the structure in Figure 3(a) for analysis purpose. Here, the PTV FIR filter is represented by U time-invariant filters $H_0(z), \dots, H_{U-1}(z)$, each of length L , where $H_i(z) = \sum_{j=0}^{L-1} h_i(j)z^{-j}$.

3.2 Realizations for Decimators

Realizations for the decimator with decimation factor M shown in Figure 1(b) can be derived by transposing the realization for interpolators in Figure 3(a). The result is shown in Figure 5(a). We refer to this structure as PTV Realization for Decimators (PTVD). Here, we assume that $N = ML$, where N is the length of the interpolation filter in Figure 1(b), and L is an integer, with M and L being relatively prime. The input $x(m)$ is first filtered by the PTV FIR filter which has L coefficients with a period of M . The result $w(m)$ is then passed through a comb filter, with system function

$$C_M(z) = \sum_{i=0}^{M-1} z^{-iL} = (1 - z^{-ML}) / (1 - z^{-L}), \quad (24)$$

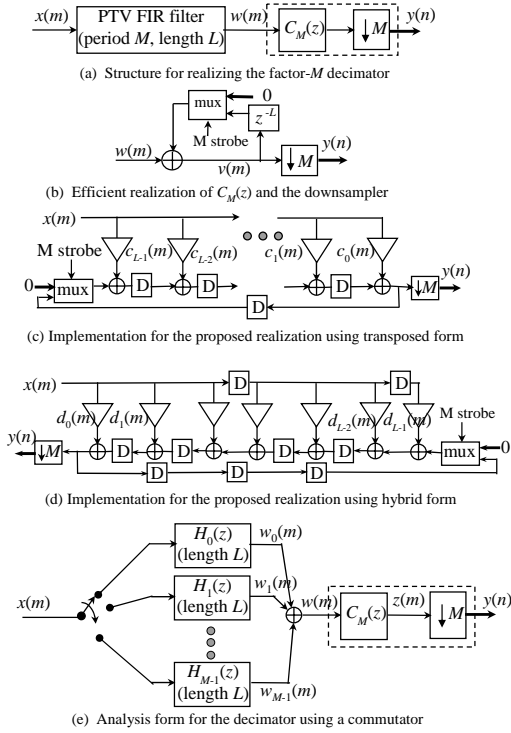


Fig. 5: The Proposed PTV Realization for a Decimator (PTVD).

and then downsampled by a factor of M to obtain the output $y(n)$.

The comb filter and downsampler $\downarrow M$ together can be implemented using either of the two structures shown in Figure 5(b). In the right-hand structure, the signal $v(m)$ is the sum of the input $w(m)$ and the signal from the multiplexer (mux). The M -strobe signal of the multiplexer selects the zero input at times $m = kM + L$, where k is an integer, and selects the signal from the feedback path at other times, i.e., the multiplexer is reset at every M -th value. Note that the signal $v(m)$ and the signal at the output of the comb filter in Figure 5(a) are not equal at every m . However, they are equal at times $m = kM$, where k is an integer, so that after the downsampler the outputs of Figures 5(a) and 5(b) are the same.

Figure 5(c) depicts the resulting realization when the PTV FIR filter uses the transposed form. In this case, the delay units of the transposed form can be shared with the feedback path of the structure in Figure 5(b). Here the multiplexer position is pushed back by $L - 1$ delays, compared to that of Figure 5(b). hence, the M -strobe signal in Figure 5(c) must select the zero input at times $m = kM + L - (L - 1) = kM + 1$, where k is an integer.

Figure 5(d) is the resulting realization when a hybrid form is used for the PTV FIR filter. As shown, each module of the hybrid form consists of 3 coefficients and three delays, with two out of the three delays being in the lower path. Here we should use as

many delay units in the lower path as can be allowed by the fan-out in the upper path, in order to maximize the number of delay units that are shared with the feedback path of the comb filter/downsampler circuit. The M -strobe signal must reset the multiplexer at proper times.

Figure 5(e) shows an alternative form for Figure 5(a), using commutator and LTI filters of length L . Here, the impulse response of the LTI filter in the p -th path be denoted by $h_p(m)$, which is zero for $m < 0$ and $m \geq L$, and $H_p(z) = \sum_{j=0}^{L-1} h_p(j)z^{-j}$ be the corresponding system function.

Now we find the impulse responses of the PTVD structure shown in Figure 5(a), using the alternative structure in Figure 5(e). To this end, we inject an impulse $\delta(m - p)$ at the input and find the output, $p = 0, 1, \dots, M - 1$. The impulse passes through the filter $H_p(z)$ in the p -th branch, resulting in an output $w_p(m)$ as

$$w_p(m) = h_p(m - p) \quad (25)$$

Since the input to all other branches are zero, we have $w(m) = w_p(m)$. The signal $w(m)$ of duration L is processed by $C_M(z)$ which repeats the signal $w(m)$ M times, yielding

$$\begin{aligned} z(m) &= \sum_{j=0}^{M-1} w(m - jL) \\ &= \sum_{j=0}^{M-1} h_p(m - p - jL) \end{aligned} \quad (26)$$

Since $h_p(m)$ has a length of L and it is zero outside the range $[0, L - 1]$, it follows that $\sum_{j=0}^{M-1} h_p(m - p - jL)$ simply repeats the values of $h_p(m)$ M times, yielding a sequence of length LM , from $m = p$ to $m = p + LM - 1$. Hence, we can write

$$z(m) = \begin{cases} h_p(\langle m - p \rangle_L), & m \in [p, p + LM - 1] \\ 0, & \text{otherwise} \end{cases} \quad (27)$$

The output signal $z(m)$ is downsampled by a factor of M to obtain

$$z(kM) = \begin{cases} h_p(\langle kM - p \rangle_L), & k \in [\lceil (p/M) \rceil, \lceil (p/M) \rceil + L - 1] \\ 0, & \text{otherwise} \end{cases} \quad (28)$$

The signal $z(kM)$ is the response of PTVD due to the impulse $\delta(m - p)$ at the input. The response is equated to the target response $g_D(k, p)$ which is related to the coefficients of the target decimation filter as described in (6). Therefore, we can write

$$z(kM) = g_D(k, p) = h_D(kM - p) \quad (29)$$

Letting $m = kM - p$ in (28) and (29), we obtain

$$h_D(m) = \begin{cases} h_{\langle -m \rangle_M}(\langle m \rangle_L), & m = 0, 1, \dots, LM - 1 \\ 0, & \text{otherwise} \end{cases} \quad (30)$$

Using (10) and (30), we obtain the final relation between the PTV coefficients of PTVD and the coefficients of the target decimation filter as

$$\begin{aligned} h_D(m) &= h_{\langle -m \rangle_M} (\langle m \rangle_L) \\ &= c_{\langle m \rangle_L - \langle -m \rangle_M} (\langle -m \rangle_M) \end{aligned}$$

which is valid for $m \in [0, LM - 1]$. Since L and M are relatively prime, no two coefficients of the decimation filter would be mapped to the same PTV FIR filter coefficient with the same time, i.e., if $m_1 \neq m_2$ and $0 \leq m_1, m_2 \leq LM - 1$, then $h_D(m_1)$ and $h_D(m_2)$ will be mapped to $c_j(m)$ and $c_k(n)$ where $j \neq k$ or $m \neq n$ or both. Therefore, all the FIR-PTV coefficients $c_l(m)$ for $l = 0, 1, \dots, L - 1$ and $m = 0, 1, \dots, M - 1$ can be uniquely determined from the target filter coefficients $h_D(m)$.

Note that we assumed that $L = N/M$ and that L and M are relatively prime. If these conditions are not satisfied, we have to extend the target filter length by appending 0 coefficients until these conditions are satisfied. Note also that the number of multipliers is reduced from N in Figure 1(b) to L in Figure 5.

3.3 Realizations for Rational Multirate Converters

Next, we consider realizations for a multirate converter as shown in Figure 1(c) with a rational conversion ratio of U/M . We assume that U and M are relatively prime and that the conversion filter in Figure 1(c) has a length of N .

We proceed the derivation as follows. First, we apply the procedure used in Section 3.1 to the upsampler and the conversion filter $H_C(z)$, obtaining a structure that comprises an upsampler by U , a comb filter, and a PTV FIR filter, followed by the original downsampler by $\downarrow M$. Then the procedure used in Section 3.2 is applied to the PTV FIR filter together with the downsampler. To see how the entire process works, we use an example.

Consider a rate conversion by a factor of $2/3$, i.e., $U = 2$ and $M = 3$. Suppose that the conversion filter has a length of 130, as shown in Figure 6(a). Since $130 = 2 \times 65$ and 2 and 65 are relatively prime, we can apply the method in Section 3.1 yielding the block diagram of Figure 6(b), consisting of a factor-of-2 upsampler, a comb filter $C_2(z)$, a PTV FIR filter of period 2 and filter length of 65, and a factor-of-3 downsampler. Next, the PTV FIR filter is represented by the commutator form in Figure 3(e) and the downsampler is pushed backward through the adder, yielding Figure 6(c). Note that each branch of Figure 6(c) consists of a time-invariant filter $H_i(z)$, $i = 0$ or 1, followed by a downsampler $\downarrow 3$, constituting a decimator. Therefore, each branch can be realized using the method given in Section 3.2 i.e., it can be realized by a PTV FIR filter of period 3 and filter length L , followed by comb filter $C_3(z)$ and factor-of-3 downsampler. Since 65 is not divisible by 3, we increase

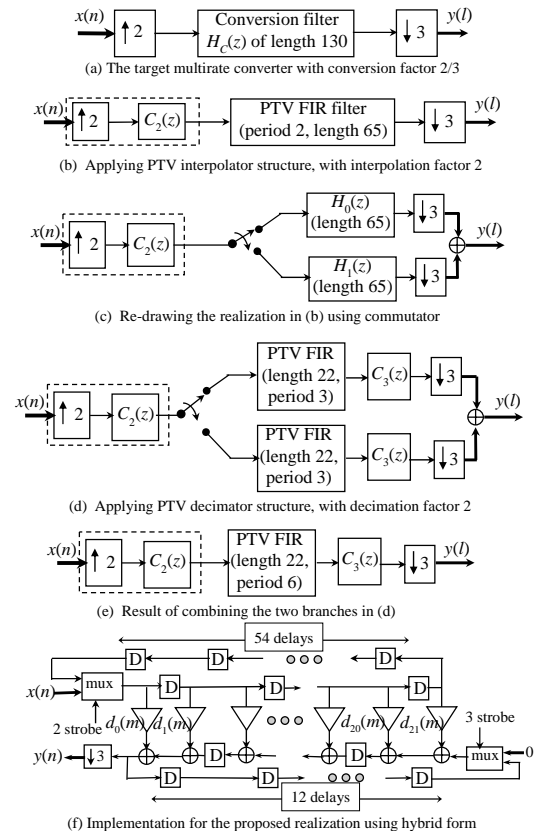


Fig. 6: An example of PTV realization for a rational rate converter.

the filter length of $H_i(z)$ to 66, so that $L = 66/3 = 22$. Since 3 and 22 are relatively prime, there is no need to further increase the value of L . Now, if we apply the structure of Figure 5(e), we arrive at Figure 6(e). The final filter is a PTV FIR filter with period 6 and length of 22. The number of multipliers needed is 22, which is about or 6 times less than the original conversion filter of length 130. Using the hybrid form for the PTV FIR filter, we obtain Figure 6(f).

Comparing a direct realization of Figure 6(a) and the proposed realization Figure 6(f), the proposed one uses only 22 multipliers, 86 delay units, and two multiplexers, while a direct realization requires 130 multipliers and 129 delay units.

The derivation above applies the PTVI structure first followed by the PTVD structure. The orders in applying these two structures may be reversed, which may yield a result with different amount of hardware.

4. EXAMPLES

In [14][15], computationally efficient designs for interpolation and decimation were presented. We select a few examples and analyze three different designs for interpolators, namely, one stage interpolation filter design, one stage filter of Martinez and Parks [14], and the interpolation filter proposed by Sarimaki [15].

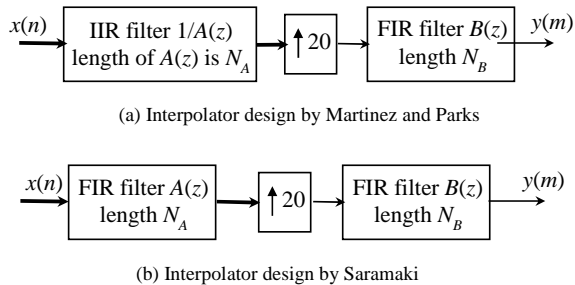


Fig. 7: Example of interpolator realization

Consider an interpolator with interpolation factor of $U = 20$, passband edge of $f_p = 0.0225$, stopband edge of $f_s = 0.025$, passband ripple $\delta_p \leq 0.05$ and stopband ripple $\delta_s \leq 0.005$.

The length of target FIR filter which meets these requirements is $N = 653$, estimated using the formulas of Hermann et al.[16]. We extend the target filter length to $N' = 660$ for implementations based on polyphase structure and PTVI. The PTV filter of the PTVI structure has $L = 33$ with a period of 20. The required hardware for implementing this filter using transposed form, polyphase structure, and PTVI structure with hybrid PTV filter, are shown in Table 1. Also given are the critical path delay and the signal rate of the multiply-and-add (MAC) operation. Here τ_{MAC} denotes the delay of a two-input multiply-and-add unit, τ'_{MAC} denotes the delay of a three-input multiply-and-add unit, and f_x and f_y denote the signal rates of the input $x(n)$ and the output $y(m)$, respectively.

We can see that the PTVI structure has minimum hardware with a similar processing speed as the conventional realization using the transposed form. The polyphase structure has a lower MAC signal rate requirement (or equivalently a higher processing speed) with similar amount of hardware as the conventional realization using the transposed form.

Now consider the structure proposed by Martinez and Parks [14] for the specifications above. It uses the following type of transfer function.

$$H_{MP}(z) = \frac{B(z)}{A(z^{20})} \quad (31)$$

Let us use N_A and N_B to denote the lengths of $A(z)$ and $B(z)$, respectively. It is found that the design specifications are satisfied with $N_A = 5$ and $N_B = 117$. The filter is realized as a cascade of recursive and non-recursive sections as depicted in Figure 7(a). The non-recursive part of the filter can be implemented using transposed form, polyphase structure, or PTVI structure. The required hardware, critical path delay, and the signal rate of a MAC unit are shown in Table 1. The same conclusion as the case of conventional filter can be made regarding the tradeoff between speed and hardware.

Next we consider the structure proposed by Saramaki[15]. It uses a cascade of transfer functions as follows:

$$H_S(z) = B(z)A(z^{20}) \quad (32)$$

Filter lengths of $N_A = 34$ and $N_B = 114$ satisfy the design specifications. It is realized as a cascade of two recursive sections, one section ($A(z)$) before the upsampler and one section ($B(z)$) after the upsampler, as shown in Figure 7(b). The filter section $B(z)$ along with the upsampler can be realized using the direct form, polyphase, and PTVI structures. The hardware, critical path delay, and MAC signal rate are shown in Table 1. Again the same conclusion as above can be made.

Therefore, we can conclude from Table 1 that the proposed structure provides significant reduction in hardware with similar speed as the conventional transposed form, while the polyphase has the advantage of a faster processing speed (or a slower requirement of MAC unit) with similar hardware as the conventional transposed form. The table also indicates that the Martinez-Parks structure is the most economical and efficient for implementation since it makes use of an all-pole section. However, if we restrict ourselves to only FIR filters, the Saramaki design is best among the three.

If the interpolation filter, the decimation filter, or the conversion filter is designed using multiplier-free coefficients, such as coefficients which are powers of two or sums of powers of two [17]-[19], then the proposed realizations will not have any multiplier and the number of adders is reduced by a factor of approximately U , M , or UM , respectively.

5. CONCLUSION

In this paper, efficient realizations for multirate filters were proposed, based on periodically time-varying (PTV) structures. Due to the periodicity of the system, the same hardware is re-used over a period to perform computations related to different sets of coefficients. This is possible due to computational redundancy of multirate filters. The proposed realizations use fewer filter elements compared to a direct-form realization, while retaining the same filtering speed. Specifically, we implemented a multirate FIR filter of length N by a PTV FIR filter of length $\frac{N}{U}$ or $\frac{N}{M}$, where U is the the upsampling factor of the interpolator and M is the downsampling factor of the decimator. In the case of a rational sampling rate conversion of U/M , we can combine the proposed structures for interpolator and decimator to obtain hardware reduction by a factor of approximately UM .

The proposed realizations provide alternatives to polyphase structures for multirate structures. Polyphase realizations reduce the signal rates at which the filters operate, while the new structures

Table 1: Hardware for various implementations of the interpolator in the example

Filter type	Implementation	Filter length	Critical path delay	MAC signal rate	Number of multipliers/adders/delays
Conventional filter	Transposed form	$N = 653$	τ_{MAC}	f_y	653/652/652
	Polyphase	$N' = 660$ (33 per path)	τ_{MAC}	$f_x = f_y/20$	660/640/640
	PTVI (hybrid form)	$N' = 660$ $L = 33$, period=20	τ'_{MAC}	f_y	33/32/65
Martinez-Parks Interpolator	Transposed form	$N_A = 5, N_B = 117$	τ_{MAC}	f_y	122/121/120
	Polyphase	$N_A = 5, N'_B = 120$ (6 per path)	τ_{MAC}	$f_x = f_y/20$	125/105/104
	PTVI (hybrid form)	$N_A = 5, N'_B = 140$ $L = 7$ and period=20 for $B(z)$	τ'_{MAC}	f_y	12/11/17
T. Saramaki Interpolation Filter Design	Transposed form	$N_A = 34, N_B = 114$	τ_{MAC}	f_y	148/147/146
	Polyphase	$N_A = 34, N'_B = 120$ (6 per path)	τ_{MAC}	$f_x = f_y/20$	154/134/133
	PTVI (hybrid form)	$N_A = 34, N'_B = 140$ $L = 7$	τ'_{MAC}	f_y	41/40/46

operate at the same signal rate as a direct-form realization, but with less hardware. Through some examples, we observed that the hardware saving is significant. Hence, the proposed structures can serve as efficient realizations for multirate filters.

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