Joint Optimal Resource Allocation and PAPR Reduction Algorithm for OFDMA Systems

Pattama Phoomchusak¹ and Chaiyod Pirak², Non-members

ABSTRACT

One of the drawbacks of an orthogonal frequency division multiplexing (OFDM) system is the larger peak to averaged power ratio (PAPR). The PAPR reduction scheme is a complicated problem, especially when a resource allocation is considered jointly. In this paper, a joint algorithm for determining an optimal resource allocation with PAPR awareness is proposed. Firstly, an adaptive tone reservation assignment is performed by optimally choosing a group of subcarriers, whose channel gains are sufficiently high, to be the data subcarriers and the remaining subcarriers will be used as the reserved tones for reducing PAPR to the desired level. Secondly, an adaptive power adjustment for PAPR reduction technique is introduced for determining the allocated power of data subcarriers by using the water-filling approach with the joint PAPR and capacity constraints. By using the proposed joint algorithm, the PAPR threshold level is satisfied while the desired capacity could be achieved. From the results, the proposed algorithm achieves the better results compared with a conventional partial transmit sequence (PTS) technique and the tone reservation (TR) technique in terms of PAPR, and probability of error performances, in which the side information is not required, in contrast with the other techniques.

Keywords: PAPR, Tone Reservation, OFDM, Resource Allocation

1. INTRODUCTION

The OFDM has been chosen for high data rate communications due to considerable high spectral efficiency, multipath delay spread tolerance, immunity to the frequency selective fading channels and power efficiency. Thanks to its advantages, the OFDM is an interesting option for high data rate wireless sensor network, Digital Video Broadcasting (DVB), mobile worldwide interoperability for microwave access (mobile WiMAX) and the future broadband radio system (5G) [1]. However, a major drawback in OFDM systems is the high peak-to-average power ratio (PAPR) of transmitting signals due to the superposition of many subcarriers, in which the high dynamic ranges is required for a nonlinear device such as a power amplifier (PA) to avoid the amplitude clipping of the signal [2]. Different approaches have been designed to cope with the PAPR problem. An amplitude clipping technique has been proposed in [3], in which the baseband signal from IFFT is clipped to the predefined threshold in the time domain. The amplitude clipping is a simplest technique, but it could generate both in-band and out-of-band distortions. For a block coding technique proposed in [4], the usefulness of this technique is limited to multicarrier systems with a small number of subcarriers, and the required exhaustive search for a good code is computationally expensive. A selected mapping technique (SLM) proposed in [5] and a partial transmit sequence technique (PTS) proposed in [6] can reduce the PAPR by controlling the phase of data subcarriers, which provides an effective solution. However, they require a transmission of continuously side information to the receiver, which degrades the capacity of the system. A tone reservation (TR) technique proposed in [7] and [8] is one of the most effective techniques when the number of subcarrier is large. In this technique, a portion of subcarriers (tone), not being used for data transmission, are reserved to create a dummy data in time domain which can be used to minimize the PAPR of the overall signal. Note that, since the dummy data on the reserved tone is separated (in frequency domain) from the data subcarriers, the data is not distorted and, hence, the probability-of-error performance will not be degraded. However, there is no rule for the tradition TR to define the number and position of reserved tones.

In [9], an adaptive TR with subcarriers allocation awareness has been proposed. A non-selected tone resulted from the optimal subcarrier allocation algorithm, based on the maximum subcarrier’s channel gain approach, will be used as a reserved tone. This technique could enhance the capacity while reducing PAPR of the OFDMA systems. However, an optimal number of reserved tones are not investigated. In [10], a cross-layer design between a TR-PAPR reduction and a subcarrier allocation algorithm is proposed for determining the optimal number of reserved tones to achieve the throughput requirement. In [11], the joint optimization of subcarrier allocation and TR

¹² The authors are with Sirindhorn International Thai-German Graduate School of Engineering (TGGS), King Mongkut’s University of Technology North Bangkok, Bangkok, Thailand, E-mail: patphoom@tot.co.th and chaiyod.p.ce@tggs-bangkok.org
PAPR reduction technique based on a fairness constraint for all users is proposed. However, the power allocation does not be mentioned in this technique. It is obvious that the existing techniques are investigated based on the equal power distribution for all of data subcarriers, which may not be valid in the realistic systems. Recently, some theoretical approaches to determine the power distribution have been proposed. The water-filling policy is a well-known technique to determine the required minimum power of subcarriers by using the channel gain and the data rate condition [12]. However, the different transmission powers allocated to different subcarriers could affect the performance of PAPR reduction, as shown in [13-14]. Different techniques have been designed to minimize the PAPR by adjusting subcarrier power levels. In [15], the simple PAPR reduction technique by using the input sequence envelope scaling approach is proposed by modifying amplitude envelope of non-overlapping groups of subcarriers with different weights in the frequency domain in which the PAPR is reduced. However, the capacity performance awareness is not mentioned in this technique. For [16-18], the PAPR reduction technique are proposed based on the power variance approach, the dynamically extension of the active constellation points (ACE) approach, and the subcarrier power window adjustment, respectively.

The previous techniques of the resource allocation and PAPR are optimized individually. In order to design an efficient algorithm to jointly optimize the resource allocation while constraining the PAPR to a certain threshold, a new algorithm for multiple objective optimizations has to be investigated. This is the motivation of this paper to propose a joint optimal resource allocation and PAPR reduction algorithm aiming at decreasing the PAPR level given the optimum power distribution and capacity constraints of the OFDMA systems. The contribution of this paper includes the adaptive power adjustment technique and the resource allocation and PAPR reduction algorithm, which can be jointly optimized for practical application of OFDMA downlink systems; meanwhile, the side information regarding the power level adjustment is not needed to transmit to the receiver. Therefore, an additional processing is not required at the receiver side.

The organization of this paper is as follows: We introduce the system model in section 2, and PAPR reduction techniques for OFDM systems in section 3. The resource allocation scheme for OFDM systems is described in section 4. In section 5 and 6, the proposed scheme and its simulation results are presented, respectively. The conclusion is given in section 7.

2. SYSTEM MODEL

In this section, we describe a system model used in this research, regarding orthogonal frequency division multiplex (OFDM) signal and PAPR problem. After the modulated symbol $X_n$ at the $n$th subcarrier in the frequency domain is passed through a serial-to-parallel converter, this frequency component will be converted to the time domain signal at the $k$th sample point by IFFT processing in order to generate an OFDM signal. The baseband sample for OFDM signals, with $N$ subcarriers, at the output of IFFT is given by [1],

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi nk/N}.$$  \hspace{1cm} (1)

The OFDM sample after adding a cyclic prefix is then ordered by the parallel-to-serial converter, and converted to the analog signal by the digital-to-analog converter (D/A). Before transmitting to the channel, the baseband OFDM signal is passed through a nonlinear power amplifier (PA). The peak power of the OFDM signal could be distorted when passing through the D/A converter or PA, resulting in a substantial amount of signal distortion. The PAPR of the signal is defined as [1],

$$PAPR(dB) = 10 \log \left( \frac{\max_{0 \leq k \leq N-1} |x_k|^2}{E[|x_k|^2]} \right).$$  \hspace{1cm} (2)

where $E[\cdot]$ is the expectation operator, representing the averaged power of the signal.

3. PEAK-TO-AVERAGE POWER RATIO REDUCTION TECHNIQUES

In this section, a summary of PAPR reduction techniques is reviewed, including a tone reservation technique, a partial transmission sequence technique, and an input sequence envelop scaling technique.

3.1 Tone Reservation Technique

The tone reservation (TR) technique can reduce the PAPR value by utilizing the reserved subcarriers, which are not used for data transmission. Based on the TR technique, the baseband signal in (1) is modified as,

$$x_k = \frac{1}{\sqrt{N}} \left( \sum_{d \in D} X_d e^{j2\pi kd/N} + \sum_{r \in R} C_r e^{j2\pi kr/N} \right).$$  \hspace{1cm} (3)

where $D$ is the set of subcarriers used for the data transmission, and $R$ is the set of remaining subcarriers used for the reserved tones. $X_d$ and $C_r$ stand for the modulated data signal on the $d$th subcarrier and the dummy data signal on the $r$th subcarrier, respectively. Basically, $C_r$ must be chosen in order to minimize the maximum value of the time-domain signal $x_k$. Then, a minimax PAPR optimization problem is defined as [7],
\[
C_r^{(\text{opt})} = \arg \min_{c_r} \max_{0 \leq k \leq N-1} |x_k|^2.
\]

One of the simple suboptimal techniques to define the value of \(C_r\) without the frequency band expansion is the iterative clipping and filtering (ICF) [2]. In each of the iteration, the baseband signal obtained from IFFT is clipped by a limiter to the predefined threshold in the time domain [3]. The clipped OFDM signal, \(x_{\text{clipped}}\), becomes [7]

\[
x_{\text{clipped}} = \begin{cases} 
  x_k, & |x_k| \leq A \\
  Ae^{j\phi_k}, & |x_k| > A 
\end{cases}
\]

where \(A\) is the clipping level and \(\phi_k\) is the phase of \(x_k\). The \(x_{\text{clipped}}\) is converted to frequency domain and then filtered such that the clipping noise exists only on the reserved tones. In this approach, the frequency domain signal is only changed at the reserved tone locations.

### 3.2 Partial Transmission Sequence Technique

For a typical partial transmission sequence (PTS) approach, the input data block in \(X\) is partitioned into \(Y\) disjoint sub-blocks, which are represented by the vectors \(X_y = [X_y, 0, X_y, 1, \ldots, X_y, N-1]\), where \((0 \leq y \leq Y - 1)\) and \(N\) is the total number of symbols. Then, the sub-blocks \(X_y\) are transformed to \(Y\) time-domain partial transmit sequences by IFFT. These partial sequences are independently rotated by phase factors \(b = \{b_y = e^{j\theta_y}, y = 0, 1, \ldots, Y - 1\}\) in order to obtain the time domain OFDM signals with the lowest PAPR. It can be defined as [6],

\[
x' = \sum_{y=0}^{Y-1} b_y x_y.
\]

The search complexity is depended on the number of allowed phase factors \(W\). Hence, \(W^{Y-1}\) sets of phase factors are searched to find the set of phase factors.

### 3.3 Input Sequence Envelope Scaling Technique

The idea of the envelope scaling technique is to scale the envelope of the input in some subcarriers in order to obtain the minimum PAPR at the output of IFFT. First, the input data block \(X\) is partitioned into \(Z\) disjoint sub-blocks or clusters, which are represented by the vectors \(\{X_z, z = 0, 1, \ldots, Z - 1\}\). All vectors \(X_z\) are scaled with the scaling factors \(s_z e(0,1], z = 0, 1, \ldots, Z - 1\) and combined together before being sent to the IFFT. For single scaling factor approach, only \(Z\) iterations have been done. Finally, the scaling factors are chosen to minimize the PAPR of \(x'\) [14], where

\[
X' = \sum_{z=0}^{Z-1} s_z X_z.
\]

\[
x' = IFFT\{X'\}.
\]

### 4. SUBCARRIER AND OPTIMAL POWER ALLOCATION TECHNIQUES

In this section, a summary of an adaptive subcarrier allocation and water-filling technique are presented.

#### 4.1 Subcarrier Allocation Technique

For the frequency-selective fading channel in downlink OFDMA systems, the channel gains are different for each subcarrier. This characteristic is used to adaptively assign the subcarriers to all users. Assuming that there are \(U\) users and \(N\) subcarriers in the system, the maximum change in achieved data rate of the \(u\)th user can be defined by [19],

\[
V_u = \frac{B}{N} \log_2 \left( \frac{1 + p_{u,n}(H_{u,\text{mean}} + s_u) \lambda H_n^{-1}}{1 + p_{u,n}(H_{u,\text{mean}} - s_u) \lambda H_n^{-1}} \right)
\]

where \(B\) is the total bandwidth of the system. \(H_{u,\text{mean}}\) is the average channel gain for all subcarriers of the \(u\)th user, and \(s_u\) is the \(u\)th user's channel gain standard deviation from the mean.

#### 4.2 Power Allocation Technique

For the water-filling technique, the spectrum can be considered as a vessel and the shape of the bottom of this vessel is determined by the inverse of \(H_n\) values, \(H_n^{-1}\). The additional power, which is the distance between the water level and the \(H_n^{-1}\) of each subcarrier, can be defined as follows [20],

\[
P_n = \begin{cases} 
  \lambda - H_n^{-1}, & \forall n : 1 \leq n \leq N \\
  0, & \text{otherwise}.
\end{cases}
\]

where \(\lambda = 2^{R_{\text{req}}/N} \prod_{n=1}^{N} (H_n^{-1})^{1/N}\) is a water-filling level.

### 5. THE PROPOSED JOINT OPTIMAL RESOURCE ALLOCATION AND PAPR REDUCTION ALGORITHM

As mentioned in the literature, the existing subcarrier and power allocation does not consider the PAPR problem. In this section, we propose the joint optimal resource allocation and PAPR reduction (JORP) algorithm design for OFDMA systems. Basically, the goal of the proposed algorithm is to minimize the PAPR of OFDM systems by using the TR technique with the capacity constraint, \(R_{\text{req}}\). Specifically, the
PAPR will be minimized by optimally inserting the specific complex signal $C_r$, determined by the TR technique, in the $r$th reserved subcarriers with the capacity constraint, expressed as follows,

$$ C_r^{(opt)} = \arg \min_{C_r} \max_{0 \leq k \leq N-1} |x_k|^2, \text{where} $$

$$ x_k = \frac{1}{\sqrt{N}} \left( \sum_{d \in D} X_d P_d e^{j2\pi kd/N} + \sum r \in R C_r e^{j2\pi kr/N} \right) $$

Subject to

$$ R_{eq} \geq \frac{B}{N} \sum_{d \in D} \log_2 \left( 1 + \frac{P_d |H_d|^2}{N_0 (B/N)} \right) $$

where $D$ is the set of subcarriers used for the data transmission, and $R$ is the set of remaining subcarriers used for the reserved tones. In addition, $X_d$ denotes the modulated data signal of the $d$th data subcarrier with the allocated power $P_d$, $H_d$ denotes the channel’s frequency response of the $d$th data subcarrier, and $B$ is the total bandwidth of the system. Since the explicit solution for an optimal $C_r$ and $P_d$ achieving a minimal PAPR with the capacity constraint is not trivial, we propose a hybrid recursive search algorithm, called JORP, which consists of two suboptimal algorithms, including an adaptive tone reservation assignment algorithm and an adaptive power allocation algorithm with PAPR awareness. The structure of downlink OFDMA system with the proposed JORP algorithm is illustrated in Figure 1.

![Fig.1: The Structure of Downlink OFDMA Systems with the Proposed JORP Algorithm.](image)

The proposed algorithm is described as follow.

The following notations are used in the JORP algorithm:

- $B$ is the total bandwidth of the OFDMA system
- $P_{APR_{th}}$ is PAPR threshold used as the performance benchmark of the system
- $Q = \{1, 2, \ldots, U\}$ is a set of user, $U = 2^i, i \in \mathbb{Z}^+$ where $U$ denotes the number of user in the system
- $D$ is a set of data subcarrier,
- $R = \{1, 2, \ldots, N\}$ is a set of reserved subcarrier, $N = 2^i, i \in \mathbb{Z}^+$ where $N$ denotes the number of subcarriers
- $C_r$ is dummy data for the $r$th reserved subcarrier of the $n$th subcarrier of the $u$th user
- $s_u$ is the $u$th user’s channel gain standard deviation with respect to the mean
- $Z$ is the number of subcarriers for each user
- $X_d$ is the data symbol of the $d$th data subcarrier
- $X_{\text{clipped}}$ is the clipped OFDM signal in frequency domain
- $x_k$ is baseband OFDM signal at the $k$th sample point of IFFT processing where $k = \{1, \ldots, N\}$
- $x_{\text{clipped}}$ is the clipped OFDM signal in time domain
- $t$ is the number of ICF iteration
- $\phi_k$ is the phase of $x_k$
- $\phi_\ell$ is the initial decision level
- $\lambda$ is a water-filling level constant
- $\beta_\ell$ is a weight factor
- $p_{\text{aw}}^{(i)}$ is the allocated power of the $w$th non-modified subcarrier at the $i$th decision level
- $p_{\text{am}}^{(i)}$ is the allocated power of the $m$th modified subcarrier at the $i$th decision level
- $X_w$ is the data symbol of $w$th subcarrier
- $X_{\text{aw}}^{(i)}$ is the data symbol at $n$th subcarrier
- $X_{\text{am}}^{(i)}$ is baseband OFDM signal at the $k$th sample point of IFFT for the $i$th decision level and $k = \{1, \ldots, N\}$, where $N$ denotes the total number of subcarriers
- $x_{\text{aw}}^{(i)}$ is the data signal of the $d$th subcarrier in frequency domain at the $i$th decision level
- $\phi_k$ is the phase of $x_k^{(i)}$
5.1 An Adaptive Tone Reservation Assignment Algorithm (ATR)

For the JORP algorithm, the ATR algorithm is performed firstly in order to assign subcarriers adaptively for achieving the PAPR threshold. The ATR algorithm is described below.

ATR Algorithm
Start:
1: \( Z = N/U \),  
2: Do  
3: \( Q = \{1, 2, \ldots, U\} \),  
4: \( R = \{1, 2, \ldots, N\} \),  
5: \( D = \emptyset \), where \( \emptyset \) denote an empty set  
6: while \( Q \neq \emptyset \)  
7: \( u^* = \arg \max_{1 \leq u \leq U} \left( B \log_2 \left( 1 + \frac{\left| \mathbb{E}[H_u,n] \right|^2}{1 + \left( \left| \mathbb{E}[H_u,n] \right|^2 - 2 \right)} \right) \) \), where \( \mathbb{E}[H_u,n] \) denote a mean of \( H_u,n \) over all subcarriers  
8: for \( m = 1 \) to \( Z \)  
9: \( n = \arg \max_{1 \leq n \leq N} |H_{u,n}|^2 \),  
10: \( D = D \cup \{n\} \) and \( R = R - \{n\} \),  
11: end for  
12: \( Q = Q - \{u^*\} \),  
13: end while  
Step 2: PAPR reduction by ICF technique  
14: if \( R \neq \emptyset \) then  
15: \( C_r = 0 \), \( \forall r \in R \)  
16: \( x_k = \frac{1}{\sqrt{N}} \left( \sum_{d \in D} X_d e^{j 2 \pi k d / N} + \sum_{r \in R} C_r e^{j 2 \pi k r / N} \right) \forall k \)  
17: for \( m = 1 \) to \( R \)  
18: \( x_{\text{clipped}, k} = \begin{cases} x_k & |x_k| \leq A \\ 0 & \text{otherwise} \end{cases} \), \( \forall k \)  
19: \( X_{\text{clipped}} = \text{FFT}(x_{\text{clipped},k}) \), \( \forall k \)  
20: \( C_r = X_{\text{clipped},r} \), \( \forall r \in R \)  
21: \( x_k = \frac{1}{\sqrt{N}} \left( \sum_{d \in D} X_d e^{j 2 \pi k d / N} + \sum_{r \in R} C_r e^{j 2 \pi k r / N} \right) \forall k \)  
22: end for  
23: \( \text{PAPR(dB)} = 100 \log \left( \frac{\max_{0 \leq k \leq N-1} |x_k|^2}{E[|x_k|^2]} \right) \),  
24: end if  
25: \( Z = Z - 1 \),  
26: while \( \text{PAPR} > \text{PAPR}_{\text{th}} \)  
The ATR algorithm can be described completely as follows.

Step 1: Subcarrier allocation  
- The maximum change in the achieved data rate of each user, \( V_u \), is measured by using (9) based on the equal power distribution in order to define the sensitivity of users. Generally, the most sensitive user has a small immunity to the frequency selective fading channels because its variance of subcarrier’s channel is very high [19], (line 7)  
- This weakest user, who has the highest \( V_u \), is prioritized as the first user to select subcarriers, which has a highest channel gain, for data transmission. (line 7)  
- The subcarrier allocation will be continuously performed until achieving the desired number of data subcarriers for each user. There are a number of unused subcarriers, which can be dedicated to be the reserved tones for reducing the PAPR value, and then this user will be eliminated from the user set. (line 8-11)  
- Likewise, this subcarrier allocation procedure is performed iteratively for all users. Based on this approach, the total capacity is maximized, and the rate proportionality among all users is maintained [19]. (line 12-13)

Step 2: PAPR reduction by ICF technique  
- The iterative clipping and filtering (ICF) technique, which is the simplest technique, is adopted in order to determine \( C_r \), inserted in the reserved tone. (line 14-22)  
- The PAPR value is then calculated and compared with the desired PAPR level, \( \text{PAPR}_{\text{th}} \). If the achieved PAPR is higher than the target, the subcarriers will be reallocated again, in which the number of reserved subcarriers will be increased in order to gain the PAPR reduction performance. (line 23-25)  
- This algorithm will be performed iteratively until achieving the desired PAPR value, and then passed through the power allocation process. (line 26)

Based on the adaptive TR used in the ATR algorithm, not only the PAPR target can be satisfied, but also subcarriers for data transmission are allocated efficiently compared with the traditional TR technique.

5.2 An Adaptive Power Allocation Algorithm with PAPR Awareness

From the ATR algorithm, one could observe that a certain number of subcarriers being used as the reserved tones can minimize the PAPR value at the expense of reducing the capacity efficiency. To cope with such problem, the conventional water-filling technique is adopted in order to allocate the power for all data subcarriers for achieving the capacity requirement.

Step 1: Unequal power distribution by the traditional water-filling technique for achieving \( R_{\text{req}} \)  
By applying the water-filling algorithm, the typical baseband OFDM signal in (3) can be modified as,

\[
x_k = \frac{1}{\sqrt{N}} \left( \sum_{d \in D} X_d P_d e^{j 2 \pi k d / N} + \sum_{r \in R} C_r e^{j 2 \pi k r / N} \right)
\] (12)
where \( P_d = \left( 2^{R_{eq}/n(D)} \prod_{d=1}^{D} (H_{d}^{-1})^{1/n(D)} \right) - H_{d}^{-1}, \forall d \)

where \( d \in D \), is the optimum power allocated for all data subcarriers, and \( n() \) denotes the total number of elements in a finite set, specified in the argument ( ). This step is illustrated in line 1-2 of the adaptive power allocation algorithm with PAPR awareness shown below.

However, the different power distribution, resulted from the water-filling technique, could impact on the amplitude variation of the signals leading to the severity of the PAPR problem. As a result, the PAPR threshold level could not be satisfied. It implies that the solution of the problem cannot be found by using only the ATR and the conventional water-filling algorithm. Therefore, a better PAPR reduction technique is required for effectively achieving the target. Actually, the existing PAPR reduction technique, such as the PTS technique and the envelope scaling technique, can be applied for the PAPR reduction purpose. However, a transmission of phase information to the receiver is needed for the PTS technique; meanwhile, the envelope scaling technique does not achieve the capacity requirement. Therefore, this motivates us to design an adaptive power adjustment for PAPR reduction (APP) technique to determine the optimal power allocation of a data subcarrier in order to reduce the PAPR value while maintaining the capacity constraint of the OFDM system without any additional side information requirement. The proposed APP technique is described as below.

**Step 2: Adaptive power adjustment for PAPR reduction (APP) technique**

As mentioned in the literatures, a power variance of data sequence is a parameter that plays an important role in PAPR reduction in the OFDM signal. Therefore, the main idea of the APP technique is that we use an exceeded power level as a function of subcarrier’s power, which is higher than a predefined level, and multiply it by a weight factor in such a way that the power variance is reduced in order to minimize the PAPR value of the overall OFDM signal. Note that the phase of the data is not affected by the APP technique; therefore, it does not need any side information or any receiver operation.

The data baseband signal allocated with an unequal power distribution for all data subcarriers in (12), expressed similarly as , is modified by the APP technique. The modified OFDM signal, \( x_{k} = IFFT\{X + C\} \), is written as

\[
x'_{k} = IFFT\{X' + C\}. \tag{13}
\]

\( X' \) is the transmitting-data signal in frequency domain, which can be defined as

\[
X' = \begin{cases} 
X_{m}P_{m} & , |P_{d}| > 1 \\
X_{w}P_{w} & , otherwise.
\end{cases} \tag{14}
\]

where \( X_{w} \) and \( X_{m} \) denote the data symbol of the \( w \)th non-modified subcarriers and the \( m \)th modified subcarriers, respectively. Based on the APP algorithm, the power amplitude of the data subcarriers is only adjusted; therefore, the data symbol \( X_{w} \) and \( X_{m} \) is still the same as the original one \( X_{d} \). \( P_{w} \) denotes the power of the \( w \)th non-modified subcarriers, which is equal to \( P_{d} \), while \( P_{m} \) denotes the modified power of the \( m \)th subcarrier, which is \( P_{m} = \beta P_{d} \), where \( \beta \) denotes a weight factor. The decision levels can be denoted as \( l \) utilized for classifying the exceed power subcarriers that would be modified by the weight factor in such a way that the variation of the data subcarriers’ amplitude is decreased, \( \beta \in [0.5,1] \). The decision levels can be obtained from

\[
l^{i} = l^{(i-1)} - \Delta, \tag{15}
\]

\[
\Delta = \frac{1}{l} \max(Pd) - \frac{1}{n(D)} \sum_{d \in D} P_{d} \right). \tag{16}
\]

where \( i \) denotes the decision level index, \( i = 1,2,\ldots,I \), and \( I \) is the total number of the different decision levels. \( \Delta \) is the distance between the different decision levels. The maximum value of \( P_{d} \) is used as an initial value of the decision levels, \( l^{(0)} \). The data subcarriers in the set \( D \) of the ATR algorithm, which occupy the power level higher than the considered decision level, are modified by \( \beta \) according to (14). This scheme is executed iteratively until the last decision level is performed. As a result, a set of different candidate signals is generated in the time domain, which represents the same data information with the different data power. In other words, I IFFT operation is needed for each data block and there are \( I \times O \) alternative representations for the different candidate OFDM signals, where \( O \) denotes the number of \( \beta \) to be allowed. Finally, the signal that has the lowest PAPR value is selected for transmission.

Although, the PAPR value can be minimized efficiently by this approach, the capacity could be degraded due to the reduction of power by a factor of \( \beta \). Hence, it would be developed for achieving the capacity requirement by modifying the data signal in frequency domain in (14) as,

\[
X' = \begin{cases} 
X_{m}P_{m} & , |P_{d}| > 1 \\
X_{w} \left( P_{w} + \frac{1}{n(D)} \sum_{d \in D} (1-\beta)P_{d} \right) & , otherwise.
\end{cases} \tag{17}
\]

Based on this approach, the power loss resulted from the weight-modification process is compensated by adding the same amount of the lost power to the remaining data subcarriers equally. As a result, the capacity could be improved approximately to the original level. Consequently, the modified baseband signal in (12) can be written as,
\[
\Delta_k^j = \frac{1}{\sqrt{N}} \left( \sum_{w \in W} X_w \cdot P_w \cdot e^{j2\pi k w / N} + \sum_{m \in M} X_m \cdot P_m \cdot e^{j2\pi k N / N} \right)
\]
(18)

where \( W \) and \( M \) denotes the set of non-modified subcarriers and modified subcarriers, respectively. Note that \( D = M + W \).

In summary, the APP technique can be described as follows, see also the adaptive power allocation algorithm with PAPR awareness.

- An initial value of the decision levels, \( l^{(0)} \) is defined by the maximum value of \( P_d \). (line 3-4)
- The distance between the different decision levels (\( \Delta \)) is determined. (line 5)
- The data subcarriers in the set \( D \) of the ATR algorithm, which occupy the power level higher than the considered decision level, are modified by \( \beta \) according to (17). (line 9-16)
- The PAPR value of the candidate OFDM signal is then calculated. (line 17-19)
- This scheme is executed iteratively with the next decision levels \( l^{(i)} \) until the last one is performed. As a result, a set of different candidate signals is generated in the time domain. (line 20)
- Finally, the signal that has the lowest PAPR value is selected for transmission.

By using the proposed APP technique, the PAPR of the OFDM signal with different power allocation for all subcarriers can be reduced while the capacity performance could be maintained unlike the envelope scaling technique. Another benefit of the APP technique is that the computational complexity of the APP technique for achieving the lowest PAPR signal is lower than that of the partial transmit sequence (PTS) techniques comparatively. Furthermore, any side information regarding the power adjustment is not required by the receiver, in contrast with the PTS technique. Moreover, it can be further applied to the traditional PAPR reduction technique easily such as the TR-ICF technique, the PTS technique and the envelope scaling technique in order to improve the PAPR reduction performance.

Step 3: PAPR additional improvement by ICF technique

Finally, if the lowest PAPR signal obtained from the APP technique is not achieved by comparing with the PAPR threshold, the traditional ICF technique can be further applied to gain the PAPR reduction performance by determining \( C_r \), filled in the reserved tone. The ICF algorithm will be performed iteratively until completing the predefined number of ICF iteration, or satisfying the PAPR threshold. This step is shown in line 22-31 of the adaptive power allocation algorithm with PAPR awareness described below.

The Adaptive Power Allocation Algorithm with PAPR Awareness

Start:

Step 1: Unequal power distribution by the traditional water-filling technique for achieving \( R_{eq} \)

1: \( \lambda = 2 R_{eq} / n(D) \sum_{d \in D} (H_d^{-1}) / n(D) \), where \( n(D) \) denotes the total number of elements in a finite set, specified in the argument ( ), and \( D \) denotes a product operator

2: \( P_d = \lambda - H_d^{-1} \),

Step 2: The APP technique

3: \( \beta = \arg\max_{d \in D} P_d \),

4: \( \lambda = P_d \),

5: \( \Delta = \frac{1}{2} [ P_d - \overline{P_d} ] \) where \( \overline{P_d} = E[P_d] \) denotes a mean of \( P_d \) over all data subcarriers

6: for \( i = 1 \) to \( I \)

7: \( l^{(i)} = l^{(i-1)} - \Delta \), where \( l^{(i)} \) denotes the iterative index at the \( i \)th decision level

8: \( W_i = D \) and \( M_i = \emptyset \), where \( \emptyset \) denotes an empty set

9: for \( d = 1 \) to \( n(D) \)

10: if \( P_d \geq l^{(i)} \) then

11: \( W_i = W_i - \{ d \} \) and \( M_i = M_i \cup \{ d \} \),

12: \( \beta = \beta P_d \), where \( m \in M_i \)

13: end if

14: end for

15: \( P_m^{(i)} = P_m^{(0)} + \frac{1}{m(w)} \left( \sum_{d \in M_i} l - \beta P_d \right) \), where \( w \in W_i \)

16: \( X_m^{(i)} = X_m P_m^{(i)} + X_m P_m^{(i)} \cdot C_r \),

17: \( x_k^{(i)} = \text{IFFT} \{ x_k^{(i)} + C_r \}, \forall k \)

18: \( C_r = 0, \forall r \) where \( r \in R \)

19: \( \text{PAPR}^{(i)} = 10 \log \left( \frac{\max_{\forall k \in K \cup N} |x_k^{(i)}|^2}{E |x_k^{(i)}|^2} \right) \)

20: end for

21: \( i^* = \arg\min_{i} \text{PAPR}^{(i)} \), \{ \( x_k^{(i^*)} \) is selected \}

Step 3: PAPR additional improvement by ICF technique

22: for \( a = 1 \) to \( \text{tot} \)

23: \( x_{\text{clipped}} \left\{ x_k^{(i^*)}, \right. \left. \left| x_k^{(i^*)} \right| \leq A \right\} \left. \left( A \cdot e^{j\theta_k}, \right. \left. \left| x_k^{(i^*)} \right| > A \right\}, \forall k \}

24: \( X_{\text{clipped}} = \text{FFT} \{ x_{\text{clipped}}, \forall k \} \)

25: \( C_r = X_{\text{clipped}, r}, \forall r \) where \( r \in R \)

26: \( x_k^{(i^*)} = \text{IFFT} \{ X_k^{(i^*)} + C_r \}, \forall k \)

27: \( \text{PAPR}^{(i^*)} = 10 \log \left( \frac{\max_{\forall k \in K \cup N} |x_k^{(i^*)}|^2}{E |x_k^{(i^*)}|^2} \right) \)

28: if \( \text{PAPR}^{(i^*)} \geq \text{PAPR}_{th} \) then

29: break,

30: end if

31: end for

The complexity of the ATR algorithm can be evaluated by counting the total number of operation (multiplication) per round in a recursive loop [19],
as follows. Let $U$ denotes the total number of user in the system, and $N$ denotes the number of subcarriers. The subcarrier gain $H_{u,n}$ for each user $u$ requires the $NU((1 + N)/2)$ operations, thus the complexity is $O(UN^2)$, in order to determine the data and reserved subcarriers. Next, the ICF process is required iteratively for determining the dummy signal $C_r$, filled in the reserved subcarriers with $O(N)$ operations. Therefore, the complexity of the ATR algorithm is $O(UN^2)$. For the adaptive power allocation algorithm with PAPR awareness, the water filling repeatedly operates $U$ times for distributing the power into the data subcarriers with the complexity of $O(U)$ operations. Therefore, the complexity of the JORP algorithm is $O(UN^2)$. It is worth noticing that the proposed JORP algorithm offers a good result in the sense of the PAPR reduction and not noticing that the proposed JORP algorithm offers a good result in the sense of the PAPR reduction and the capacity at the expense of higher computational complexity.

6. SIMULATION RESULTS

This section presents computer simulation results to verify the performance of the proposed algorithm in terms of the PAPR reduction, capacity, and bit error rate performances for a single-cell downlink OFDM-based multiple access systems. In this paper, we assume that the BPSK modulation with 64 subcarriers is used for 2 users in the 1 MHz bandwidth systems. A six-independent frequency selective Rayleigh fading multipath channel model with the typical urban with exponential power profile (COST 231) is modeled. The perfect knowledge of the channel state information (CSI) for all users is assumed to be known at the base station. The PAPR reduction is evaluated in terms of its complementary cumulative density function (CCDF) that is the probability that PAPR exceeds a given the desired threshold $\gamma$; (CCDF = Prob (PAPR > $\gamma$)). In our simulation, 5000 different channel realizations are used, and the results are averaged. The subcarrier channels for each user are generated following the Jake’s model [1], and the average SNR is ranged from 1-40 dB. The simulation parameters used in the following evaluations are expressed as follows.

1) We assume that the PAPR threshold in the simulation is $6.5$ dB at CCDF of 10-2, which is less than the minimum PAPR for the 802.11 a/g WLAN standard [21].

2) To simplify the comparison in the simulation, the capacity requirement is assumed to be 2.2 bits/s/Hz (at SNR = 10 dB), which is sufficiently high enough comparing with the maximum capacity for the 802.11 a/g WLAN standard [21].

3) Basically, the computational complexity of all technique depends on the disjoint group of subcarrier, which is modified by the factor. In this simulation, the total number of the disjoint decision levels $I$ is 4 for the APP technique, and the input data block is partitioned into 4 disjoint clusters for both PTS and envelope scaling techniques.

The JORP algorithm is firstly processed by the ATR operation for determining the number and position of reserved tones that achieve the PAPR threshold. The PAPR reduction resulted from the ATR algorithm by using the equal power distribution for all data subcarriers, is shown in Figure 2.

It is worth noticing that the PAPR reduction by using 16 iteration of ICF technique with 8 reserved tones is about $7.6$ dB at CCDF of 10-2, which is not achieve the PAPR target in contrast with using 14 reserved tones. However, the number of reserved tones directly reduces the capacity shown in Figure 3. Specifically, it can be seen that using 14 reserved tones provide the capacity of 1.8 bits/s/Hz at SNR = 10 dB, which is lower than using 8 reserved tones by 0.5 bits/s/Hz at SNR = 10 dB. From these results, the subcarrier assignment, which is solely based on the PAPR target criterion, could not satisfy the desired capacity. To cope with such problem, the adaptive power allocation algorithm with PAPR awareness is then performed. Firstly, the traditional water-filling technique is applied for allocating the different power to the data subcarriers given the capacity requirement, which is 2.2 bits/s/Hz (at SNR = 10 dB). The PAPR reduction by using the TR-ICF technique comparing between the equal power distribution case and the unequal power distribution case obtained from the traditional water-filling technique is shown in Figure 4. From the figure, the PAPR level of the unequal power distribution case can be reduced increasingly from 9 dB to 6.9 dB at CCDF of 10-2 by using 16 iteration of ICF technique. However, it is worse than that of the equal power distribution case about 0.4 dB at CCDF of 10-2. From this result, we have found that the different power distribution affects the PAPR reduction performance, even when applying the TR-ICF technique for the PAPR reduction.

Subsequently, the APP technique is introduced for reducing the PAPR value by adjusting the power of some data subcarriers, which have the power level higher than the considered decision level $l$, with a multiplicative factor $\beta$. The experimental result of the trade-off curve between the PAPR and the capacity with different $\beta$ values of the APP technique is shown in Figure 5. Basically, the power variance of the data subcarriers is minimized according to $\beta$ value, which leads to PAPR reduction. Therefore, we could observe that PAPR is greatly reduced when $\beta$ is decreasing. At $\beta = 0.5$, the lowest PAPR is reached, which is about 4.5 dB; whereas the capacity is most degraded, which is about 4.3 bits/s/Hz. However, the capacity is also depending on an amount of subcarriers’ modified power, which are classified by the
decision level. Therefore, $\beta = 0.5$ is used in the following simulation because it provides the lowest PAPR experimentally. A trade-off curve between the PAPR and the capacity with the different decision levels at $\beta = 0.5$ is shown in Figure 6. From this figure, the decision level $l^{(6)}$ provides the lowest PAPR, which is about 4.5 dB whereas the highest PAPR at the decision level $l^{(1)}$ is about 6.3 dB. However, the capacity at the decision level $l^{(6)}$ is worse, which is about 4.3 bits/s/Hz, due to a large amount of data power is decreased for the PAPR reduction propose. From these results, we have found that the PAPR reduction performance increases when the decision level tends to be decreasing; meanwhile, the capacity is degraded gradually. However, the power compensation strategy is adopted in the APP technique to deal with the degradation of the capacity performance.

The PAPR reduction of the JORP algorithm is shown in Figure 7. By using the total number of the different decision levels $1 \leq 4$, hence, 4 IFFT iterative operations are executed for each data block in order to obtain the lowest PAPR level for the APP scheme. From this figure, the JORP algorithm achieve the PAPR target value (6.5 dB at CCDF of $10^{-2}$) by converges at 3 additional ICF iterations; meanwhile, the PAPR level of the water-filling distribution case is reduced from 9 dB to 7 dB at CCDF of 10-2 by the traditional TR-ICF technique with 16 iterations. From this result, it implies that solely applying the TR-ICF technique cannot achieve the target value even when applying 16 iterations. Therefore, it is possible to combine the ICF and APP techniques to make the convergence of the traditional TR much faster. Note that, the PAPR reduction of the TR-ICF technique proportionally depends on the number of reserved tones in contrast with the JORP algorithm. Therefore, the PAPR reduction performance of the JORP algorithm could be better than the TR-ICF technique notably in the system that uses a small number of reserved tones.

The capacity performance comparison of the JORP, the water-filling and the equal power distribution case by using the TR-ICF technique with 16 iterations is shown in Figure 8. From this figure, the capacity performance of the JORP algorithm is nearly close to that of the water-filling distribution by 2.2 bits/s/Hz at SNR of 10 dB, which is the capacity requirement. This is due to the JORP algorithm employs a power-penalty approach for the unmodified data subcarriers equally. Certainly, the capacity requirement could not be achieved by distributing equal power for all data subcarriers. By using the JORP algorithm, the capacity requirement and the PAPR target could be jointly achieved.

In comparison to other techniques, the standard PTS technique and the envelope scaling technique are applied to the unequal power distribution signal in a frequency domain for reducing the PAPR level. To be fair in the process of adjusting the phase or amplitude for all techniques, the scaling factors $\{0.5, 1\}$ are used in the envelope scaling technique, and the phase factors $\{1, -1\}$ are used in the PTS technique. For the PTS technique, the partial sequences are independently rotated by phase factors after IFFT process in order to obtain the time domain OFDM signals with the lowest PAPR. Therefore, the computational expense is further required by 8 iterations for searching the optimum set of phase factors unless 4 IFFT operations. As a result, the PTS technique yields higher computational complexity than that of both the envelope scaling and the APP techniques, needing only 4 IFFT iterative operations. The PAPR reduction performance comparison is illustrated in Figure 9. From the figure, the PTS technique and the envelope scaling technique can reduce the PAPR level from 9 dB to 7 dB and 8 dB at CCDF of $10^{-2}$ respectively, which could not achieve the PAPR threshold level unlike the JORP algorithm. From the result, it is clear that the PAPR reduction performance of the JORP technique is better than that of both PTS and envelope scaling technique by 0.5 dB and 1.5 dB at CCDF of $10^{-2}$, respectively.

In Figure 10, the investigation of the capacity performance for these techniques is illustrated. As a result of the power-compensated procedure in the JORP algorithm, the capacity performance of the JORP algorithm is almost close to that of the water-filling technique. It imply that the JORP algorithm achieve the capacity requirement approximately. Moreover, the capacity of the PTS technique is also equal to the capacity requirement because the phase of data subcarriers is only changed without any power distribution disturbance. For the envelope scaling technique, which is based on the amplitude level reduction without the compensation scheme, the capacity performance is worse than the requirement about 0.5 bits/s/Hz. The bit error rate (BER) performance comparison in the non-linear channel is investigated in Figure 11. We could observe that the BER of the JORP algorithm yields less distortion than that of the TR-ICF technique with 16 iterations, the PTS technique and the envelope scaling technique for a given SNR.

The PAPR reduction performance of the JORP algorithm comparing to the TR-ICF technique, the traditional PTS technique and the envelope scaling techniques based on the 8QAM modulation is shown in Figure 12. From the figure, we could observe that the PAPR in the case of 8QAM is larger than that of BPSK. Nonetheless, the JORP algorithm can reduce the PAPR value of the unequal power distribution from 11 dB to 6.5 dB at CCDF of $10^{-2}$, which is better than that of the TR-ICF technique with 16 iterations, the traditional PTS technique and the envelope scaling techniques by 1.5 dB 2 dB and 2.5 dB at CCDF of $10^{-2}$, respectively. Last but not least, the
PAPR reduction performance of the JORP algorithm for BPSK, QPSK, and 8QAM modulation with the different number of subcarrier are shown in Figure 13. From the figure, we could observe that the PAPR in the case of 8QAM is larger than that of both BPSK and QPSK when the number of subcarrier tends to be increasing. Moreover, the PAPR reduction gains are obtained by using the JORP algorithm in different modulation scheme.

7. CONCLUSION

In this paper, a joint optimal resource allocation and PAPR reduction algorithm, called the JORP algorithm, has been proposed. It consists of two suboptimal algorithms, which are an adaptive tone reservation assignments algorithm used for achieving the PAPR target, and an adaptive power allocation algorithm with PAPR awareness used for achieving the capacity constraint. Specially, the APP technique, which is applied in the JORP algorithm, is newly designed for reducing the exceeded PAPR value of the unequal power distribution of the water-filling operation without capacity degradation. Based on the JORP algorithm, the optimal allocated power for data subcarriers are investigated for reducing the PAPR to the threshold value while the capacity could be achieved approximately to the requirement of OFDMA systems. From the simulation results, the JORP algorithm can reduce the PAPR level from 9 dB to 6.5 at CCDF of $10^{-2}$, which is better than that of the TR-ICF technique, the PTS technique and the envelope scaling technique by 0.5 dB, 0.5 dB and 1.5 dB at CCDF of $10^{-2}$, respectively. Another benefit of the JORP algorithm is that none of the side information is required by the proposed technique in contrast with the PTS technique. It is worth noticing that the proposed JORP algorithm offers a good result in the sense of the PAPR reduction, the capacity, and probability of error performances at the expense of higher computational complexity, which is $O(UN^2)$. Finally, the JORP algorithm can be easily applied to both PTS and envelope scaling techniques in various modulation types with different size of subcarriers.

References


Fig. 7: The PAPR Reduction Performance by the JORP Algorithm.

Fig. 8: The Capacity vs. SNR of the JORP Algorithm.

Fig. 9: The PAPR Reduction Performance by the JORP Algorithm.

Fig. 10: The Capacity Performance Comparison.

Fig. 11: BER Performance Comparison.

Fig. 12: The PAPR Reduction Performance Comparison of the 8QAM Modulation.

Fig. 13: The PAPR Reduction of the JORP Algorithm for BPSK, QPSK, and 8QAM Modulation.

Pattama Phoomchusak received the B.Eng and M.Eng degree in Telecommunication Engineering from King Mongkut’s Institute of Technology Ladkrabang, Thailand, in 2000 and 2004, respectively. Currently, she is a Ph.D. candidate in wireless communication engineering laboratory of The Sirindhorn International Thai-German Graduate School of Engineering (TGGS), King Mongkut’s University of Technology North Bangkok, Thailand.
Chaiyod Pirak received the B.Eng. degree in Telecommunication Engineering in 2000 from King Mongkut’s Institute of Technology Ladkrabang, Thailand. In 2005, he received the Ph.D degree in Electrical Engineering from Chulalongkorn University, Thailand in association with University of Maryland College Park, USA. In 2009, he was appointed as the head of mobile communications and embedded systems laboratory in the department of electrical and software system engineering, The Sirindhorn International Thai-German Graduate School of Engineering (TGGS). At present, he is a lecturer at communication engineering department of TGGS. His research are in wireless sensor network; embedded system, FPGA, DSP, and microcontroller; broadband wireless access: WiMax, WCDMA, and CDMA2000; digital signal processing for wireless communications.