The Method for Determining Parameters of Approximate 2DOF Digital Controller for Robust Control of DC-DC Converter

Eiji Takegami¹, Kohji Higuchi², Kazushi Nakano³, Satoshi Tomioka⁴, and Kazushi Watanabe⁵, Non-members

ABSTRACT

Robust DC-DC converters which can cover extensive load changes and also input voltage changes with one controller are needed. In this paper, we propose the method for determining the parameters of 2DOF digital controller which makes the control bandwidth wider, and at the same time makes variation of the output voltage very small in spite of sudden changes of the resistive load and the input voltage. The 2DOF digital controller whose parameters are determined by the proposed method is actually implemented on a DSP and is connected to a DC-DC converter. Experimental studies demonstrate that this type of digital controller can satisfy given specifications.

Keywords: DC-DC Converter, DSP, Digital Control, Robust Control, Approximate 2-Degree-Of-Freedom System

1. INTRODUCTION

In many applications of DC-DC converters, loads cannot be specified in advance, i.e., their amplitudes are suddenly changed from the zero to the maximum rating. Generally, design conditions are changed for each load and then each controller is re-designed. Then, a so-called robust DC-DC converter which can cover such extensive load changes and also input voltage changes with one controller is needed. In DC-DC converters used for devices, such as the latest FPGA etc., the requirements not only to dynamic load responses but start-up responses are hard in order to protect restrictions of the standup sequence etc., Analog control ICs are used usually for the controller of DC-DC converters. Simple integral control and related functions etc. are performed with the analog control ICs. Moreover, the applications of the digital controllers to DC-DC converters designed by the PID or root locus method etc., has been recently considered[1, 2]. However, it is difficult to retain sufficient robustness of DC-DC converters by these classical 1-degree-of-freedom (1DOF) techniques. For example, in the reference [1], only the start-up response is thought as important, and the overshoot is little but the rising time is not so short and the big ripple has arisen at the steady state. And in the reference [2], the start-up response and the dynamic load response are taken into consideration. However, since a 1DOF controller is used, the big over-shoot arises at the start-up response and the rising time is long, and the big voltage change has arisen at the dynamic load response. Some simulations and experiments show that this new DC-DC converter can satisfy given specifications.

The authors proposed the method of designing an approximate 2-degree-of-freedom (2DOF) controller of DC-DC converter[3]. Since a second-order model was used for a reference model of 2DOF system, the controller became more complex. In this paper, we propose a method for determining the parameters of 2DOF digital controller used a first-order model[4, 5]. This controller makes the control bandwidth wider, and at the same time reduces considerably a variation of the output voltage at sudden changes of resistive load and the input voltage. The parameter of the controller is determined based on the view of obtaining sufficient approximation of 2DOF digital controller and obtaining sufficient low sensitivity. In order to obtain sufficient approximation, it is required to set up the target characteristics arbitrarily. Therefore, the estimated current feedback is used with the voltage feedback. The new DC-DC converter equipped with the proposed controller in DSP is made experimentally. Some simulations and experiments show that this new DC-DC converter can satisfy given specifications.

2. DC-DC CONVERTER

The DC-DC converter as shown in Fig.1 has been made experimentally. In order to realize the approximate 2DOF digital controller which satisfies given specifications, we use the DSP(TI TMS320LF2401). This DSP has a built-in AD converter and a PWM switching signal generating part. The triangular wave
carrier is adopted for the PWM switching signal. The switching frequency is set at 300[KHz] and the peak-to-peak amplitude $C_m$ is 66[V]. The LC circuit is a filter for removing carrier and switching noises. $C_0$ is 308[$\mu F$] and $L_0$ is 1.4[\mu H].

If the frequency of control signal $u$ is lower than that of the carrier, the state equation of the DC-DC converter at a resistive load in Fig.1 except for the controller in DSP can be expressed from the state equalizing method[6] as follows:

\[
\begin{align*}
\dot{x} &= A_c\bar{x} + B_c u \\
y &= C\bar{x}
\end{align*}
\]

(1)

where

\[
\bar{x} = \begin{bmatrix} e_0 \\ i \end{bmatrix}, \quad A_c = \begin{bmatrix} -\frac{1}{C_m L_0} & -\frac{1}{C_m T_0} \\ -\frac{1}{C_m} & -\frac{1}{C_m L_0} \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ \frac{V_i N_2}{C_m N_1} \end{bmatrix}
\]

\[
C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad u = e_i, \quad y = e_o, \quad K_p = -\frac{V_i N_2}{C_m N_1}
\]

and $R_0$ is the total resistance of coil and ON resistance of FET, etc., and its value is 0.015[\Omega]. When realizing a digital controller by a DSP, a delay time exists between the starting time of the sampling operation and the outputting time of the control signal due to the calculation and AD/DA conversion times. This delay time is considered to be equivalent to the input dead time which exists in the controlled object as shown in Fig.2. The system of Fig.2 is the new controlled object. The element $D_{Ld}$ is input dead time $L_d$ and the element Z.O.H is zero order hold. The element $(1/\tau)$ is one sampling delay used for current estimation. New additional state variables are $\xi_1$ and $\xi_2$, and the new control input is $v$. $T$ is the sampling period. Then the discrete-time state equation of the new controlled object shown in Fig.2 is expressed as follows:

\[
\begin{align*}
\begin{bmatrix} x_{d\dot{w}}(k+1) \\ y(k) \end{bmatrix} &= A_{d\dot{w}} x_{d\dot{w}}(k) + B_{d\dot{w}} v(k) \\
y(k) &= C_{d\dot{w}} x_{d\dot{w}}(k)
\end{align*}
\]

(2)

where

\[
x_{d\dot{w}}(k) = \begin{bmatrix} x_d(k) \\ \xi_2(k) \end{bmatrix}, \quad \bar{x}_d(k) = \begin{bmatrix} x(k) \\ \xi_1(k) \end{bmatrix}, \\
A_{d\dot{w}} = \begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix}, \quad B_{d\dot{w}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
A_d = e^{A_c T} e^{A_c (T-L_d)} \int_0^{L_d} e^{A_c \tau} B_c d\tau, \\
A_d = \begin{bmatrix} A_d(1,1) & A_d(1,2) & A_d(1,3) \\ A_d(2,1) & A_d(2,2) & A_d(2,3) \end{bmatrix}, \\
B_d = \begin{bmatrix} \int_0^{T-L_d} e^{A_c \tau} B_c d\tau \\ 1 \end{bmatrix}, \\
C_{d\dot{w}} = \begin{bmatrix} C_d & 0 \end{bmatrix}, \quad C_d = \begin{bmatrix} C \\ 0 \end{bmatrix}, \quad \xi_1(k) = u(k)
\]

Here, $A_d$, $B_d$ and $C_d$ are derived from the system surrounded by the dotted line in Fig.2.

In practical use of a DC-DC converter, the characteristics of a startup transient response, a dynamic load response and an output response at input voltage sudden change are important. The DC-DC converter with the following specifications 1-7 is designed and manufactured:

1. Input voltage $V_i$ is 48[V] and output voltage $e_o$ is 3.3[V].
2. Startup transient responses are almost the same at resistive load and parallel load of resistance and capacity, where $0.165 \leq R_L < \infty$[\Omega] and $0 \leq C_L \leq 200[\mu F]$. $T$ is the sampling period.
3. For the specs. 2, 3, 4 and 5, the system is stable and the reference input and the parameter changes are bounded, the equivalent disturbances are bounded[7]. The parameter changes are transformed into the equivalent disturbances $q_e$ and $q_o$ as
shown in Fig.3. Therefore, what is necessary is just to constitute the control systems whose pulse transfer functions from the equivalent disturbances \( q_v \) and \( q_y \) to the output \( y \) become as small as possible in their amplitudes, in order to robustize or suppress the influence of these parameter changes and input voltage change.

\[
W_{ry}(z) = \frac{(1 + H_1)(1 + H_2)(1 + H_3)}{(z - n_1)(z - n_2)(z + H_1)} \times \frac{(z - n_1)(z - n_2)(z + H_4)}{(z + H_2)(z + H_3)(z + H_4)} \tag{3}
\]

where \( n_1 \) and \( n_2 \) are the zeros for the discrete-time controlled object (2). It shall be specified that the relation of \( H_1 \) and \( H_2, H_3 \) becomes \(|H_1|>|\text{Re}(H_2)|, |H_1|>|\text{Re}(H_3)|\). Then \( W_{ry}(z) \) can be approximated to the following first-order model:

\[
W_{ry}(z) \approx W_m(z) = \frac{1 + H_1}{z + H_1} \tag{4}
\]

This target characteristic \( W_{ry}(z) \approx W_m(z) \) is specified so that it satisfies the specs.3 and 4.

Applying a state feedback

\[
v = -Fx^* + GH_2 r
\]

\[
x^* = [y \ x_2 \ \xi_1 \ \xi_2]^T
\]

and a feedforward

\[
\xi_1(k+1) = Gr
\]

into the discrete-time controlled object as shown in Fig.3, we decide \( F = [F(1, 1) F(1, 2) F(1, 3) F(1, 4)] \) and \( G \) so that \( W_{ry}(z) \) becomes eq.(3). In Fig.3, \( r \) is the reference input and \( \eta(k) = \xi_1(k + 1) \). Current feedback is used in Fig.3. This is transformed into voltage and control input feedbacks, without changing the pulse transfer function between \( r - y \) by an equivalent conversion. The following relation is obtained from Fig.3:

\[
-F(1, 2)x_2(k) = \frac{4}{A_d(1, 2)} \left( x_1(k + 1) - A_d(1, 1)x_1(k) - A_d(1, 3)\xi_1(k) - B_d(1, 1)\eta(k) \right) \tag{7}
\]

If the current feedback is transformed equivalently using the right-hand side of this equation, the control system with only voltage feedback as shown in Fig.4 will be obtained. The transfer function \( W_{Qy}(z) \) be-

\[
W_{Qy}(z) = \begin{bmatrix} W_{q_v}(z) & W_{q_y}(z) \end{bmatrix} \tag{8}
\]

between this equivalent disturbance \( Q = [q_v \ q_y]^T \) and \( y \) of the system in Fig.4 is desfined as

\[
W_{Qy}(z) = \begin{bmatrix} W_{q_v}(z) & W_{q_y}(z) \end{bmatrix} \tag{8}
\]

The system added with the inverse system and filter to the system in Fig.4 is constituted as shown in Fig.5. In Fig.5, the transfer function \( K(z) \) becomes

\[
K(z) = \frac{k_z}{z - 1 + k_z} \tag{9}
\]

\( W_m^{-1}(z) \) in Fig.5 is the inverse of eq.(4), and the existence of \( W_m^{-1}(z) \) is always guaranteed and
where
\[ FF(1,1) = -A_d(1,1)/A_d(1,2) \]
\[ FF(1,2) = A_d(1,2) \]
\[ FF(1,3) = -A_d(1,3)/A_d(1,2) \]
\[ FF(1,4) = -B_d(1,1)/A_d(1,2) \]
\[ F_z = F(1,4) - F(1,2)FF(1,4) \]

3.2 Determining method of parameters of the controller

Now, for good approximation, i.e., in order to let eqs. (10) and (11) approach the right-hand side of eqs. (13) and (14) respectively, it is necessary to set up \( W_s(z) \) so that it may approach 1 in the large frequency range. Moreover, it is necessary to make the gain of eq. (14) small for low sensitivity. For these purposes, while the gain of \( W_Qy \) is small, \( k_z \) has to be set at a large value. The parameter \( k_z \) is limited within \( 0 < k_z < 1 \) in order to realize the relation \( K(z)W_m^{-1}(z) \approx W_m^{-1}(z) \). The parameter \( k_z \) is one which affects the loop gain of the system of Fig.5 greatly, and when \( k_z \) is enlarged too much within the limits, the roots of the closed loop system of Fig.5 comes out of the unit circle, and has the possibility of becoming unstable. Then, \( k_z \) has to be set as a suitable value by which the closed loop system is not made unstable. If the value of \( k_z \) is larger, the roots of the following equation (corresponding to the denominator in eq.(10)) may approach \( H_1 \), and the accuracy of approximation may become not so good.

\[ z - 1 + k_z W_s(z) = 0 \]  

(16)

The roots of eq.(16) are poles of whole systems except for \( H_1 \) and \( H_4 \). When \( k_z \) increases from 0, these roots leave 1, \(-H_2\), and \(-H_3\), and when \( k_z \) is set at a certain value, they become certain values \( p_1, p_2 \) and \( p_3 \) as shown in Fig.7. If determining \(-H_2\) and

![Fig.6: Approximate 2DOF digital integral type control system](image_url)

\[ k_1 = F(1,1 + F(1,2)FF(1,1)) + ((-F(1,4) - F(1,2)FF(1,4))(-F(1,2)/FF(1,2)) \]
\[ + (GH_4 + GF_z)(k_z/(1 + H_2)) \]
\[ k_2 = F(1,2)/FF(1,2) + G(k_z/(1 + H_2)) \]
\[ k_3 = F(1,3) + F(1,2)(FF(1,3)) k_4 = -F_z \]
\[ k_5 = Gk_z, k_2 = G(k_5 + GF_z)k_z \]
\[ k_{r1} = G k_{r2} = GH_4 + GF_z \]  

(15)
the absolute values of the real number parts of those becomes suitably small when \( k_2 \) is a sufficiently large value, the accuracy of approximation of eqs.(10) and (11) will become good, and low sensitivity will also become good.

\[
(|\text{Re}(p_1)|, |\text{Re}(p_2)|, |p_3|) \ll |H_1| \tag{17}
\]

If specifying \( p_1, p_2 \) and \( p_3 \) as in eq.(17), and \( k_2 \) to be suitable value, we can search for starting points \( H_2 \) and \( H_3 \) of root loci, in a reverse way. If the roots of eq.(16) are specified to be \( p_1, p_2 \) and \( p_3 \), the following equation will be obtained from eq.(12).

\[
(1 - n_1)(1 - n_2)(z - 1)(z + H_2)(z + H_3) + k_2(1 + H_2)(1 + H_3)(z - n_1)(z - n_2) = (z - p_1)(z - p_2)(z - p_3) \tag{18}
\]

Substituting \( H_2 = x + yi, H_3 = x - yi \) and the value of \( k_2 \) into eq.(18), and making the coefficient of each power of \( z \) equal, three circle equations with respect to \( x \) and \( y \) will be obtained. \( H_2 \) and \( H_3 \) can be determined from the intersection of these circles.

4. EXPERIMENTAL STUDIES

The sampling period \( T \) are set at 3.3[\( \mu \text{s} \)] and the input dead time \( L_d \) is about 0.999\( T[\mu\text{s}] \). The numbers of turns \( N_1 \) is 8 and \( N_2 \) is 4, so \( K_P = -0.18 \). The nominal value of \( R_L \) is 0.33[\( \Omega \)]. We design a control system so that all the specifications are satisfied. First of all, in order to satisfy the specification on the rising time of startup transient response, \( H_1 \) and \( H_4 \) are specified as

\[
H_1 = -0.89 \quad H_4 = -0.3 \tag{19}
\]

If setting up with

\[
p1 = 0.35 + 0.5i \quad p2 = 0.35 - 0.5i \quad p3 = 0.5
\]

and substituting this into eq.(18), the following circle equations are obtained:

\[
2x + 0.00000262 + 0.00000164y^2 + 0.2 = 0 \tag{21}
\]

\[
-1.696x + 1.152x^2 + 1.152y^2 - 0.570 = 0 \tag{22}
\]

\[
0.296x - 0.852x^2 - 0.852y^2 + 0.334 = 0 \tag{23}
\]

Here \( n_1 \) and \( n_2 \) are as

\[
n_1 = -0.97351 \quad n_2 = -0.97731 \times 10^6 \tag{24}
\]

These circles are drawn as shown in Fig.8. The intersections of these circles can be found as

\[
x = -0.1 \quad y = 0.6 \tag{25}
\]

Therefore, from this intersection, \( H_2 \) and \( H_3 \) are determined as

\[
H_2 = -0.1 + 0.6i \quad H_3 = -0.1 - 0.6i \tag{26}
\]

Then the parameters of controller become as follows:

\[
k_1 = -332.223 \quad k_2 = 260.57 \quad k_3 = -0.51638
\]

\[
k_4 = -0.51781 \quad k_5 = 7.0594 \quad k_6 = -8.6321 \tag{27}
\]

It must be better that \( k_{r1} \) and \( k_{r2} \) are set as 0, since the characteristics of the control system hardly change in this case.

The simulation results of the startup responses are shown in Fig.9. From the output voltage \( y = e_o \) in this figure, it turns out that the specifications are satisfied. Almost the same simulation results
as Fig.9 are obtained when the input voltage $V_i$ is changed by $\pm 20\%$. The simulation result of the dynamic load responses is shown in Fig.10. Fig.10 is the result at resistive load whose value is changed as $R_L = 0.33 \leftrightarrow 0.165[\Omega]$. Almost the same simulation result as Fig.9 can be obtained at parallel load of resistance ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$) and capacity ($C_L = 200[\mu F]$). Fig.11 shows the output response at resistive load $R_L = 0.33[\Omega]$ when input voltage changed suddenly such as 48 $\rightarrow$ 38 $\rightarrow$ 48 $\rightarrow$ 58 $\rightarrow$ 48[V]. It turns out that all the specifications are satisfied.

The produced new DC-DC converter with built-in DSP is shown in Fig.12.

![DC-DC Converter](Image)

**Fig.10:** Simulation result of dynamic load response at resistive load

This is a quarter brick size (36.8mm x 57.9mm x 8mm) one. Experimental results when the digital controller with the parameters of eq.(27) is equipped with DSP shown in Figs.13-22. Fig.13 and Fig.14 show startup responses at the resistive loads $R_L = 0.33[\Omega]$ and $R_L = 0.165[\Omega]$, respectively.

![Startup Response](Image)

**Fig.13:** Experimental result of startup response at resistive load ($R_L = 0.33[\Omega]$)

![Startup Response](Image)

**Fig.14:** Experimental result of startup response at resistive load ($R_L = 0.165[\Omega]$)

From $y = e_o$ in these figure, it turns out that the
specifications are satisfied. Fig.15 shows a startup response at parallel load of resistance $R_L = 0.33[\Omega]$ and capacity $C_L = 200[\mu F]$. Fig.16 shows a startup response at parallel load of resistance $R_L = 0.165[\Omega]$ and capacity $C_L = 200[\mu F]$.

From $y = e_o$ in these figure, it turns out that almost the same experimental results as the simulation ones in Fig.9 are obtained and the specifications are satisfied. Fig.17 and Fig.18 show startup responses at resistive load $R_L = 0.33[\Omega]$ when the input voltages are 58[V] and 38[V], respectively. It turns out that the specifications are satisfied when the input voltage $V_i$ is changed by $\pm 20\%$. Fig.19 shows the dynamic load response at resistance load ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$). It turns out that almost the same experimental results as the simulation results in Fig.10 are obtained. Although the load current changed suddenly from 20 [A] to 10 [A] or the opposite, the output voltage change is very small and is suppressed within about 50[mV]. Fig.20 shows the dynamic load response at the parallel load of resistance ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$) and capacity ($C_L = 200[\mu F]$).

Fig.21 and Fig.22 show the output responses at resistive load $R_L = 0.33[\Omega]$ when input voltage changed suddenly from 38[V] to 48[V] and from 48[V] to 58[V].

5. CONCLUSION

In this paper, the concept of controller of a DC-DC converter with the built-in DSP to attain good robustness against extensive load changes and input voltage change was given. The proposed digital controller was implemented on the DSP(TI TMS320LF2401). The new DC-DC converter with
the built-in DSP was produced. It was shown from experiments that a sufficiently robust digital controller is realizable. The characteristics of the startup transient response, the dynamic load response and the output response against sudden input voltage change were improved by using the proposed parameter determining method for an approximate 2DOF digital controller. This fact demonstrates the usefulness and practicality of our method. The future work is to design a robust digital controller to realize sufficient robustness when (LC+LC) circuits, etc., are used as filters for removal of switching and carrier noises.

References


Eiji Takegami received M. Eng. degree from Oita University, Oita, Japan in 1997. From 1997 he is an Engineer, Development and Design of switching power supply at Nippon Electric Industry Co., Ltd. (present: DENSEI-LAMBDALambdak K.K.). He is a member of IEEE, IEICE, and the Society of Instrument and Control Engineers (SICE).
Kohji Higuchi received Dr. Eng. degree from Hokkaido University, Sapporo, Japan in 1981. From 1980 he was a Research Associate at the University of Electro-Communications. From 1982 he was an Assistant Professor at the University of Electro-Communications Fukuoka. He is currently an Associate Professor in the Department of Electronic Engineering, the University of Electro-Communications, Tokyo, Japan. His interests include Power Electronics and Control Engineering. He is a member of IEEE, IEICE, IEEJ and the Society of Instrument and Control Engineers (SICE).

Kazushi Nakano received Dr. Eng. degree from Kyushu University, Fukuoka, Japan in 1982. From 1980 he was a Research Associate at Kyushu University. From 1986 he was an Associate Professor at Fukuoka Institute of Technology. He is currently a Professor in the Department of Electronic Engineering, the University of Electro-Communications, Tokyo, Japan. His interests include system identification/control and their applications. He is a member of IEEE, IEICE, IEEJ and the Society of Instrument and Control Engineers (SICE).

Satoshi Tomioka received B. Eng. degree from Tokyo Denki University, Tokyo, Japan in 1984. From 1984 he is an Engineer, Development and Design of Switching Power Supply at NEMIC-LAMBDA Ltd. (present: DENSEI-LAMBDA K.K.). He is a member of IEICE.

Kazushi Watanabe received B. Eng. degree from Tokyo University of Science, Tokyo, Japan in 1982. From 1982 he is an Engineer, Development and Design of Switching Power Supply at NEMIC-LAMBDA Ltd. (present: DENSEI-LAMBDA K.K.).