ABSTRACT

This paper presents a design of PID controller for furnace temperature control system with disturbance. Currently, PID controller has been used to operate in electric furnace temperature control system because its structure is simpler compared to others. However, the issue of tuning and designing PID controller adaptively and efficiently is still open. This paper presents an improved PID controller efficiency from tuning by Nelder Mead method. The parameters of PID controller shall be obtained from the Nelder Mead optimization procedure. Errors between desired magnitude response and actual magnitude response are calculated by using the Integral of Absolute Error (IAE). The proposed Nelder Mead based PID design method is simpler, more efficient and effective than the existing traditional methods included Ziegler-Nichols, Cohen-Coon and Direct Synthesis. Simulation result shows that the performance of PID controller using this proposed method is better than traditional methods and resistant to disturbance.

Keywords: Electric Furnace, Disturbance, PID Controller, PID Parameters, Nelder Mead Optimization

1. INTRODUCTION

Electric furnace is one of many furnaces available today. It uses electricity as its main power source to generate heat which widely uses in various industrial production processes. However, the current controller design that is popular for use with electric furnace, such as PID control [1,2], neural network [3] and adaptive fuzzy control [4-8]. The PID control design is popular and easiest way for electric furnace, but it is also a problem for the design is nonlinear system [9], time delay and disturbance. Nowadays, there are many methods for tuning PID, such as Ziegler-Nichols [10], Cohen-Coon [11], Direct Synthesis [12], Genetic algorithm (GA) [13], particle swarm optimization (PSO) [14], differential evolution (DE) [15], and multi-objective optimization algorithms [16]. All of these methods do not deliver good tuning since rise time, overshoot and settling time still occur and may be not suitable for electric furnace temperature systems.

This paper proposed Nelder Mead-based PID controller for solving these problems. It is used to determine the optimal parameters of PID controller using the calculation of Integral of Absolute Error (IAE), which is traditional method for finding the best value in form of nonlinear. After applying Nelder Mead Algorithm, then the parameters $k_p$, $k_i$ and $k_d$ are obtained. These results will be compared with traditional methods included Ziegler-Nichols [10], Cohen-Coon [11] and Direct Synthesis [12] and different disturbances.

2. PID CONTROLLER

PID controller consists of Proportional, Integral and Derivative control. Proportional control is responsible for faster enter steady state, Integral control is responsible for reducing overshoot in steady state and Derivative control is responsible for making the system more stable.

This paper introduces a single-input single-output (SISO) PID controller, which consists of PID controller $D(s)$ and controlled plant $G(s)$ are shown in Fig. 1 which is simple and effective.

![Fig.1: A control system with PID controller.](image)

where $D(s)$ is transfer function of PID Controller, $G(s)$ is transfer function of controlled plant, $r(t)$ is input signal to controlled plant, $c(t)$ is the system error, $u(t)$ is controlled input and $y(t)$ is output signal.

From Fig.1, the equation of standard PID Controller is

$$u(t) = k_p e(t) + k_i \int e(t) dt + k_d \frac{de(t)}{dt}$$ (1)

and can be written in the form of transfer function is

$$D(s) \equiv \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s} + k_d s$$ (2)
where \( U(s) \) is transfer function of controlled input, \( E(s) \) is transfer function of the system error \( e(t) \), \( k_p, k_i \) and \( k_d \) are proportional gain, integral gain and derivative gain, respectively.

From (2), PID controller can be written as

\[
E(S) = \frac{k_p}{S} + \frac{k_i}{S} + k_dS
\]

**Fig. 2: Block diagram of PID controller.**

### 3. ELECTRIC FURNACE TEMPERATURE CONTROL SYSTEM

The compositions of electric furnace temperature control system [17] are electrical furnace, controller and thermocouple which controller is used to control the temperature in electrical furnace is shown as Fig. 3.

**Fig. 3: Block diagram of electric furnace control.**

where \( r \) is input voltage, \( U \) is output voltage from controller, \( y \) is output voltage from thermocouple and \( R \) is armature resistance.

In this paper, transfer function of electric furnace is chosen as [17]

\[
G(s) = \frac{0.15}{s^2 + 1.1s + 0.2}
\]

, transfer function of a 1.5 time delay is

\[
H(s) = e^{-1.5s}
\]

Then, transfer function of electric furnace with a 1.5 time delay is

\[
G(s) = \frac{0.15}{s^2 + 1.1s + 0.2} e^{-1.5s}
\]

Approximation of (4) is

\[
H(s) = \frac{1 - 0.75s}{1 + 0.75s}
\]

Then, from (5) and (6) will be

\[
G(s) = \frac{-0.1125s + 0.15}{0.75s^3 + 1.825s^2 + 1.25s + 0.2}
\]

Hence, (7) is transfer function of electric furnace, which is used for experiment in this paper.

### 4. NELDER MEAD OPTIMIZATION FOR PID CONTROLLER

In this paper, Nelder Mead optimization is used for searching the best parameters of PID controller for use with the furnace temperature control system. This method had been introduced by Nelder and Mead in 1965. It is a basic principle for determining minimum of nonlinear multiple variable equations.

Structure of control system by using Nelder Mead Optimization for PID controller is shown in Fig. 4.

**Fig. 4: Structure of Nelder Mead with a control system and PID Controller.**

In this paper, the result of the optimization is based on the error from the calculation of IAE. Result is shown in (8) and (9), which is based on the desired magnitude response and the actual magnitude response.

\[
\text{Error}(K) = f(K) = \sum_{t=0}^{n} |e(t)|, t = 0, t_s, 2t_s, \ldots, n
\]

where \( t_s \) is sampling time, \( n \) is maximum time for optimization, \( \text{Error}(K) \) or \( f(K) \) is IAE, \( K \) is parameters of PID controller, \( e(t) \) is system error, \( y(t) \) is control output or actual magnitude response and 1 is desired magnitude response.

Nelder Mead Optimization consists of B (Best point), G (Good point), W (Worse point), M (Mid...
point), E (Expansion Point), R (Reflect point), C (Construction point) and S (Shrink point).

4.1 Initial Triangle BGW

Let \( f(K) \) be the function that used for minimizing which Nelder Mead method will find the three points of a triangle as

\[
B = f(K_1), \quad G = f(K_2), \quad \text{and} \quad W = f(K_3)
\]

That B is the best point (value less than G and W), G is good point (next to best), and W is the worst point.

4.2 Mid point

The building process uses the Mid point of the line from B and G as

\[
M = \frac{B + G}{2}
\]

4.3 Expansion point

The Expansion point is calculated from Mid point and Worst point as

\[
E = 3M - 2W
\]

4.4 Reflection point

The Reflection point is calculated from Mid point and Expansion point as

\[
R = \frac{M + E}{2}
\]

4.5 Contraction point

The Contraction points that used on this paper have 2 points. The first point is calculated from Worst point and Mid point and the second point is calculated from Reflection point and Mid point as

\[
C_1 = \frac{W + M}{2} \quad \text{or} \quad C_2 = \frac{R + M}{2}
\]

4.6 Shrink Point

The Shrink point is constructed from Best point and Worst point as

\[
S = \frac{B + W}{2}
\]

All points that used for Nelder Mead method are shown as Fig.5

According to the calculation, the algorithm steps are shown as below:

1. Generate an initial configuration K randomly, where \( K_1 = [k_{p1} \ k_{i1} \ k_{d1}] \), \( K_2 = [k_{p2} \ k_{i2} \ k_{d2}] \) and \( K_3 = [k_{p3} \ k_{i3} \ k_{d3}] \).

2. Calculate \( f(K_1), f(K_2), f(K_3) \) for finding B, G, W, where B < G < W.

3. Compute M, E and f(E).

4. Compare \( f(E) \) and \( f(G) \), if \( f(E) < f(G) \) replace W with E, go to step 8; else Compute R and \( f(R) \) go to step 5.

5. Compare \( f(R) \) and \( f(W) \), if \( f(R) < f(W) \) replace W with R go to step 6.

6. Compare \( f(R) \) and \( f(G) \), if \( f(R) >= f(G) \) Compute C and \( f(C) \) go to step 7; else go to step 8.

7. Compare \( f(C) \) and \( f(W) \), if \( f(C) < f(W) \) replace W with C go to step 8; else compute S, replace G with M and replace W with S go to step 8.

8. (8) Rearrange the B, G, W, where B < G < W and repeat step (3) until some predefined stopping criteria.

The Pseudo code of Nelder Mead is shown in Fig. 6.
where \( i \) is the iteration for optimization which sets the maximum number of iterations \( i_{\text{max}} = 100 \).

5. DESIGN EXAMPLE AND SIMULATION RESULT

The input signal \( r(t) \) that used on this section is unit step function.

\[
r(t) = \begin{cases} 
0, & t < 0 \\
1, & t \geq 0 
\end{cases} 
\tag{16}
\]

, from (16) is shown as Fig.7.

![Fig.7: The input signal \( r(t) \).](image)

The disturbance \( n(t) \) that used on this section is square wave signals from -0.1 to 0.1, -0.2 to 0.2, -0.3 to 0.3, -0.4 to 0.4, -0.5 to 0.5 and -0.6 to 0.6 that is shown as Fig.8.

![Fig.8: The disturbance \( n(t) \).](image)

5.1 Optimized PID controller design for linear system with Nelder Mead Algorithm

The transfer function of linear system is

\[
G(s) = \frac{1}{s^2} 
\tag{17}
\]

Setting the ranges of \( k_p, k_i \) and \( k_d \) are between 0 to 30, maximum time for optimization \( n = 25s \), sampling time \( t_s = 0.05s \) and maximum number of iterations \( i_{\text{max}} = 100 \).

The step response for linear system under different methods based PID controller is compared in Fig.9.

![Fig.9: The step response for linear system under different methods based PID controller.](image)

The error of step response for linear system under different methods based PID controller is compared in Fig.10.

![Fig.10: The error of step response for linear system under different methods based PID controller.](image)

The performances of these methods are evaluated by these indices including rise time, %overshoot, settling time and Error (IAE) that are shown as Table 1.

From Table 1, rise time and Error \((K)\) of Nelder Mead is smaller than Ziegler Nichols; settling time and %overshoot of Nelder Mead is close to Ziegler Nichols.

Then, the results show that the transient response and steady-state performances obtained by
Nelder Mead for linear system are better than Ziegler-Nichols [10].

**Table 1:** Comparative performance of step response for linear system under different methods.

<table>
<thead>
<tr>
<th>Performances</th>
<th>Method</th>
<th>Ziegler Nichols</th>
<th>Nelder Mead</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_p$</td>
<td>1.2024</td>
<td>29.6558</td>
<td></td>
</tr>
<tr>
<td>$k_i$</td>
<td>0.0481</td>
<td>0.4977</td>
<td></td>
</tr>
<tr>
<td>$k_d$</td>
<td>7.5075</td>
<td>28.3216</td>
<td></td>
</tr>
<tr>
<td>Rise time</td>
<td>0.2689</td>
<td>0.0699</td>
<td></td>
</tr>
<tr>
<td>%overshoot</td>
<td>1.9341</td>
<td>3.0587</td>
<td></td>
</tr>
<tr>
<td>Settling time</td>
<td>0.4391</td>
<td>0.6836</td>
<td></td>
</tr>
<tr>
<td>$\text{Error}(K_i)$, $n = 25\ s$, $t_s = 0.05\ s$</td>
<td>5.8903</td>
<td>1.9151</td>
<td></td>
</tr>
</tbody>
</table>

5.2 Optimized PID controller design for nonlinear system with Nelder Mead Algorithm

The transfer function of nonlinear system is

$$G(s) = \frac{1}{s^2 + 1}$$  \hspace{1cm} (18)

Setting the ranges of $k_p$, $k_i$, and $k_d$ are between 0 to 30, maximum time for optimization $n = 25\ s$, sampling time $t_s = 0.05\ s$ and maximum number of iterations $i_{\text{max}} = 100$.

The step response for nonlinear system under different methods based PID controller is compared in Fig.11.

**Fig.11:** The step response for nonlinear system under different methods based PID controller.

The error of step response for nonlinear system under different methods based PID controller is compared in Fig.12.

From Table 2, the results show that the transient response and steady-state performances obtained by Nelder Mead for nonlinear system are better than Ziegler-Nichols [10].

**Table 2:** Comparative performance of step response for nonlinear system under different methods.

<table>
<thead>
<tr>
<th>Performances</th>
<th>Method</th>
<th>Ziegler Nichols</th>
<th>Nelder Mead</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_p$</td>
<td>4.2059</td>
<td>0.0774</td>
<td></td>
</tr>
<tr>
<td>$k_i$</td>
<td>3.6849</td>
<td>14.7575</td>
<td></td>
</tr>
<tr>
<td>$k_d$</td>
<td>1.2001</td>
<td>14.9913</td>
<td></td>
</tr>
<tr>
<td>Rise time</td>
<td>0.4863</td>
<td>0.1465</td>
<td></td>
</tr>
<tr>
<td>%overshoot</td>
<td>47.2999</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Settling time</td>
<td>15.7495</td>
<td>0.2605</td>
<td></td>
</tr>
<tr>
<td>$\text{Error}(K_i)$, $n = 25\ s$, $t_s = 0.05\ s$</td>
<td>44.7904</td>
<td>2.2304</td>
<td></td>
</tr>
</tbody>
</table>

5.3 Optimized PID controller design for electric furnace temperature system with Nelder Mead Algorithm

In this experiment, the transfer function of electric furnace from (7) will be chosen for simulating the design of PID controller which uses Nelder Mead optimization to determine the best parameters of PID controller by setting the ranges of $k_p$, $k_i$, and $k_d$ are between 0 to 30, maximum time for optimization $n = 25\ s$, sampling time $t_s = 0.05\ s$ and maximum number of iterations $i_{\text{max}} = 100$.

The step response for electric furnace under different methods based PID controller is compared in Fig.13.

The error from step responses of electric furnace under different methods based PID controller is compared in Fig.14.

The performances of these methods are evaluated by these indices including rise time, %overshoot, settling time and Error (IAE) that are shown as Table 2.
From Table 3, rise time of Nelder Mead is close to Ziegler Nichols but smaller than Cohen-Coon and Direct Synthesis; settling time and Error(K) of Nelder Mead are smaller than Ziegler-Nichols, Cohen-Coon and Direct Synthesis; %overshoot of Nelder Mead is bigger than Direct Synthesis but smaller than Ziegler-Nichols and Cohen-Coon.

Then, the results show that the transient response and steady-state performances obtained by Nelder Mead for electric furnace are better than Ziegler-Nichols [10], Cohen-Coon [11] and Direct Synthesis [12].

5.4 Optimized PID controller design for electric furnace temperature system with disturbance

The experimental results from 5.3 showed PID controller based on Nelder Mead for Electric furnace are better than traditional methods, then this experiment presents about optimized PID controller design for electric furnace temperature systems with disturbance $n(t)$ that it is shown in Fig. 15.

$$y(t) = y_1(t) + y_2(t)$$ (19)

, and can be written in the s-domain is

$$Y(s) = Y_1(s) + Y_2(s)$$ (20)

$$Y_1(s) = \frac{G(s)D(s)}{1 + G(s)D(s)} R(s)$$ (21)

$$Y_2(s) = \frac{G(s)}{1 + G(s)D(s)} N(s)$$ (22)

where $y(t)$ is the control output, $y_1(t)$ is control output from input signal, $y_2(t)$ is control output from disturbance, $D(s)$ is transfer function of PID Controller, $G(s)$ is transfer function of controlled plant, $r(t)$ is input signal to controlled plant, $e(t)$ is the system error and $u(t)$ is controlled input.

In this experiment, the transfer function of electric furnace from (7) will be chosen for simulating the design of PID controller which uses Nelder Mead optimization to determine the best parameters of PID controller by setting the ranges of $k_p$, $k_i$, and $k_d$ are between 0 to 30, maximum time for optimization $n = 25$ s, sampling time $t_s = 0.05$ s and maximum number of iterations $i_{max}=100$.

The step response of control output from input signal $y_1(t)$ of electric furnace under different disturbance presents about optimized PID controller design for electric furnace temperature systems with disturbance $n(t)$ that it is shown in Fig. 15.
The step response of control output from disturbance $y_2(t)$ of electric furnace under different disturbances based PID controller is compared in Fig.17.

Fig.17 shows that the disturbance $n(t)$ is effective only in the initial state. After the initial state, the disturbance will not affect to the control output $y(t)$.

The error of step response of control output from input signal $y_1(t)$ of electric furnace under different disturbances based PID controller is compared in Fig.18.

The step response of control output $y(t)$ of electric furnace under different disturbances based PID controller is compared in Fig.19.

From Table 4 and Table 5, they are comparative performances of Nelder Mead with low disturbance and high disturbance. The low disturbance consists of square wave signals from -0.1 to 0.1, -0.2 to 0.2 and -0.3 to 0.3. The high disturbance consists of square wave signals from -0.4 to 0.4, -0.5 to 0.5 and -0.6 to 0.6 that the performances from rise time, %overshoot and settling time is not much different but Error ($K$) varied according the increased of disturbance.

From Table 4 and Table 5, the comparative performances of Nelder Mead with disturbances, the proposed controller can well operate although the electric furnace system exists the disturbances in the system process.

5.5 Comparison PID controller design for electric furnace temperature system with very high disturbance

The experimental results from 5.4 showed PID controller based on Nelder Mead for Electric furnace...
with disturbance, it has shown that the proposed controller can well operate although the electric furnace system exists the disturbances in the system process.

In this experiment, the very high disturbance is square wave signal from -1 to 1 that it is shown as Fig.20.

![Fig.20: Very high disturbance.](image)

The transfer function of electric furnace from (7) will be chosen for simulating the design of PID controller which uses Nelder Mead optimization to determine the best parameters of PID controller by setting the ranges of \( k_p \), \( k_i \) and \( k_d \) are between 0 to 30, maximum time for optimization \( n = 25 \) s, sampling time \( t_s = 0.05 \) s and maximum number of iterations \( i_{max} = 100 \).

The PID parameters of the traditional methods from Table 3 will be chosen for comparison the performance of PID controller design for electric furnace temperature system with very high disturbance which shows a comparison as Fig.21.

![Fig.21: The comparison the performance of PID controller design for electric furnace temperature system with very high disturbance.](image)

6. CONCLUSIONS

In this paper, Nelder Mead based PID controller design method for Electric furnace temperature control system was simulated in MATLAB. The key operations of this method include maximum time for optimization, sampling time and maximum number of iterations that the performance of this method is depended on disturbance. The obvious advantages of the proposed approach are that 1) the transient and steady-state performances are better than the traditional methods included Ziegler-Nichols [10], Cohen-Coon [11] and Direct Synthesis [12] and 2) the proposed controller can well operate although the electric furnace system exists the disturbances in the system process.

7. ACKNOWLEDGEMENT

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References


Vunlop Sinlapakun received the B.Eng degree in Telecommunication Engineering from King Mongkut’s Institute of Technology Ladkrabang (KMITL), Bangkok, Thailand, in 2009 and the M.Eng degree in Electronic and Telecommunication Engineering from King Mongkut’s University of Technology Thonburi (KMUTT), Bangkok, Thailand in 2015. His research interests include control theory and applications.

Wudhichai Assawinchaichote received the B.Eng. (Hons) degree in Electrical Engineering from Assumption University, Bangkok, Thailand, in 1994; the M.S. degree in Electrical Engineering from the Pennsylvania State University (Main Campus), PA, USA, in 1997 and the Ph.D. degree with the Department of Electrical and Computer Engineering from the University of Auckland, New Zealand (2001-2004). He is currently working as a senior lecturer in the Department of Electronic and Telecommunication Engineering at King Mongkut’s University of Technology Thonburi, Bangkok, Thailand. His research interests include fuzzy control, robust control and filtering, Markovian jump systems and singularly perturbed systems.