Transmission Line Performance Indices Calculation Based on Voltage Stability Criterion

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ABSTRACT

In this paper, three performance indices of a transmission line i.e., line security margin, line security index and line severity index are defined and proposed to use for transmission planning purpose. These indices can be obtained with slight modification from the power system loadability limit calculation, which is normally limited by a static voltage stability constraint. A repetitive power flow is employed without loss of generality. Sensitivity factors of apparent power flow at the receiving end bus are adopted to predict power flow direction of each transmission line corresponding to a specified scenario of generation and demand increased. The proposed performance indices provide information regarding severe transmission paths that need for expansion or upgrading. A six-bus system is used to test with satisfactory results.

Keywords: Security Margin, Security Index, Severity Index, Loadability, Voltage Stability, Sensitivity Factors, Predictor-Corrector scheme, P-Q Plane

1. INTRODUCTION

Optimal expansion of transmission systems is a crucial issue for power system planning. The main objective is not only to provide transmission path to transfer the energy from generation to load areas. The challenge is to compromise between system reliability and costs, e.g. short term operation and investment costs [1]-[3]. In another word, it should be planned to cope with the future demand with high quality of services. However, satisfactory degree between reliability and revenue return rate from investment and operating costs is often questionable. This can be formulated as a multiobjective optimization problem and can be solved using some techniques e.g., linear programming [4] dynamic programming [5]-[6], and nonlinear programming [7]. For countries whose electric supply industry has not been fully deregulated, e.g., Thailand, generation and transmission system expansion is conducted under a Power Development Plan (PDP), which is studied and revised annually according to a long term load forecast program [8]. For a restructured system, pricing of transmission services and congestion charges cannot provide a signal to convince investors for investing in these infrastructures. In extremis, inadequate or delay of investment in new infrastructures may cause a serious event as the case of 2003 blackout in the North of US [9]-[10]. Therefore, transmission system security is a very essential issue in power system operation.

To verify security of a system, loadability limit of a power system is an index generally used to serve such purpose. Loadability limit of a power system can be normally defined as the maximum demand that can be supplied by the system corresponding to specified scenarios of increasing generation and demand, meanwhile satisfying system constraints [11]-[12]. Traditionally, continuation power flow [13]-[16], sequential power flow [17], and bifurcation theory [18]-[21] are the methodologies used to calculate the system limit. However, this calculation always limits by voltage stability criterion occurred on transmission system. Thus, adequacies of transmission paths to transfer the energy from generation to load areas play an important role to enhance loading capability of a system. However, the results obtained from the traditional methods e.g., the system load-ability limit and the weakest bus, cannot provides information regarding severity of a transmission system. Hence, they cannot be adopted for transmission planning purpose. For planning purposes, severity of a transmission system should be verified and presented through some performance indices containing useful information regarding loading capability. In addition, it is beneficial if these indices can be obtained with slightly modify from the traditional methods.

In this paper, we proposed three transmission line performance indices; line security margin, line security index and line severity index, to use for transmission planning. These performance indices can be calculated directly from a current operating point. Thus, the traditional methods e.g., a continuation or repetitive power flow and bifurcation analysis can be
used with slightly modification. The proposed indices provide information concerning loading condition of each transmission line, which is useful information for transmission expansion or upgrading study. Formulation of the proposed method adopts the voltage instability condition, presented on each transmission line P-Q plane. The method is tested with a six-bus system with satisfied results.

2. BACKGROUND THEORY

2.1 Voltage Stability Limit on a Transmission Line

Consider the equivalent π model of a transmission line connected between bus − i and bus − j as shown in Fig.1.

![Fig.1: The π model transmission line connected between bus − i and bus − j](image)

We define $V_i \angle \delta_i$ and $V_j \angle \delta_j$ as voltage magnitudes of bus − i and bus − j respectively. The series impedance is denoted by Z, whereas half of the line charging susceptance is denoted by $Yc$. The apparent power flows from bus − i to bus − j and vice versa is denoted by $P_{ij} + jQ_{ij}$ and $P_{ji} + jQ_{ji}$ respectively. For an interested line, we assume that real power flows from bus − i to bus − j. Thus, fictitious load current and apparent power at the receiving end, $r$, are $I_{ijr}$ and $P_{ijr} + jQ_{ijr}$ respectively. Obviously, the fictitious load is equal to the apparent power flowing into bus − j.

We can formulate the relationship between injected current and voltage at any buses based on generalized ABCD parameters as follows:

$$V_i \angle \delta_i = AV_i \angle \delta_j + BI_j$$  \hspace{1cm} (1)

where $A = 1 + ZYc$ and $B = Z$. The complex form of $A$ and $B$ can be expressed as shown in (2).

$$A = a_1 + ja_2 \hspace{1cm} B = b_1 + jb_2$$  \hspace{1cm} (2)

The receiving end current, $I_{ijr}$, can be expressed in (3).

$$I_{ijr} = (P_{ijr} - jQ_{ijr})/V_j \angle - \delta_j$$  \hspace{1cm} (3)

Substitute $A$ and $B$ from (2) and $I_{ijr}$ from (3) into (1) resulting in (4).

$$V_i \angle \delta_i = (a_1 + ja_2)V_j \angle \delta_j + (b_1 + jb_2)(P_{ijr} - jQ_{ijr})/V_j \angle - \delta_j$$  \hspace{1cm} (4)

Rearrange (4) resulting in (5).

$$V_iV_j = (a_1V_j^2 + b_1P_{ijr} + b_2Q_{ijr}) + j(a_2V_j^2 + b_2P_{ijr} - b_1Q_{ijr})$$  \hspace{1cm} (5)

(5) can be rewritten as shown in (6).

$$c_1V_{j+} + (c_2P_{ijr} + c_3Q_{ijr} - V_j)V_{j+} + c_4(P_{ijr} + Q_{ijr}) = 0 \hspace{1cm} (6)$$

where $c_1 = a_1z + a_2z, c_2 = 2(a_1b_1 + a_2b_2), c_3 = 2(a_1b_2 + a_2b_1)$ and $c_4 = b_1 + b_2$.

$$a(V_j^2) + b(V_j) + c = 0 \hspace{1cm} (7)$$

where $a = c_1, b = c_2P_{ijr} + c_3Q_{ijr} - V_j$ and $c_4(P_{ijr} + Q_{ijr})$.

The solution of (6) is the square of the receiving end voltage. Thus the receiving end voltage can be calculated from (8).

$$V_j = \sqrt{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$  \hspace{1cm} (8)

There are two solutions for (8) i.e., the lower solution lies on the lower part of the P-V curve and is unstable [22]. Thus, the available solution is a stable one on the upper half, which can be expressed as in (9).

$$V_j = \sqrt{\frac{-b - \sqrt{b^2 - 4ac}}{2a}}$$  \hspace{1cm} (9)

The point where the two trajectories, i.e. stable and unstable lines, are joined is the nose or bifurcation point. In addition, this is the point where the maximum power can be transferred, which is the condition in (10).

$$b^2 - 4ac = 0$$  \hspace{1cm} (10)

Substitute coefficients of the quadratic equation from (7) into (10) and rearrange, we obtain (11).

$$(c_2z - 4c_1c_4)P_{ijr}^2 + (c_3z - 4c_1c_3)Q_{ijr}^2 - 2c_2V_{j+}P_{ijr} - 2c_3V_{j+}Q_{ijr} + 2c_2c_3P_{ijr}Q_{ijr} + V_{j+}$$  \hspace{1cm} (11)

The relationship between $P_{ijr}$ and $Q_{ijr}$ of (11) is a locus of the collapsing point on the P-Q plane which separates the operating points into feasible and infeasible regions. To clarify, we rewrite (11) as shown in (12).
where the superscript \( c \) is denoted for the collapsing point and \( E_1 = (c_{22} - 4c_{12}) \), \( E_2 = (c_{22} - 4c_{12}) \), \( E_3 = -2c_2V_{i2}, \) \( E_4 = -2c_2V_{i2}, \) \( E_5 = 2c_2c_4, \) \( E_6 = V_{i2}^2. \)

In [23], for a given \( P_{ijr} \) we can calculate \( Q_{ijr} \), or vice versa, which satisfies (12) from (13) and (14) respectively.

\[
c_5(Q_{ijr})^2 + c_6Q_{ijr} + c_7 = 0 \quad (13)
\]

\[
c_8(P_{ijr})^2 + c_9P_{ijr} + c_{10} = 0 \quad (14)
\]

where

\[
c_5 = (c_{22} - 4c_{12}), \quad c_6 = 2c_2c_4 - V_{i2}^2 \]

\[
c_7 = (c_{22} - 4c_{12})(P_{ijr})^2 - 2c_2V_{i2}P_{ijr} + V_{i2}^2 \]

\[
c_8 = (c_{22} - 4c_{12}), \quad c_9 = 2c_2c_4 - V_{i2}^2 \]

\[
c_{10} = (c_{22} - 4c_{12})(Q_{ijr})^2 - 2c_2V_{i2}Q_{ijr} + V_{i2}^2 \]

Occasionally, the term \( c_5 \) or \( c_8 \) in (13) and (14) is zero. In such case, the above equations are reduced to linear form. Then \( Q_{ijr} = -c_7/c_6 \) and \( P_{ijr} = -c_{10}/c_9. \)

### 3. Calculation Framework

The proposed method consists of four main parts, i.e., in-creasing of generation and demand scenarios, gradient vector formulation, collapsing point calculation and definitions of transmission line performance indices. Details of each part are presented below.

#### 3.1 Scenarios of Generation and Demand Increased

We consider firstly a set of buses in the system which is defined as \( \Delta = \{ s, G, L \} \), where \( s \) refers to the slack bus, \( G \) is a set of generation buses, and \( L \) is a set of load buses. \( \Delta P_G \) and \( \Delta Q_G \) are defined as a set of an incremental change of real and reactive power injected at generation buses, \( \Delta P_L \) and \( \Delta Q_L \) are a set of an incremental change of real and reactive power injected to load buses. An incre-mental change of real and reactive transmission loss is de-noted by \( \Delta S_{loss}. \)

If the load change of \( \Delta P_L + j\Delta Q_L \) occurs, all the genera-tion buses must contribute their power for the consequences. This relationship can be expressed in (15).

\[
\Delta P_G + j\Delta Q_G + \Delta P_L + j\Delta Q_L + \Delta P_s + j\Delta Q_s = \Delta S_{loss} \quad (15)
\]

We define \( \lambda_n \) as a vector indicating the contribution in-dices for the increase of generation and demand as described in (16).

\[
\lambda_n = (\lambda_s, \lambda_G, \lambda_L) = (\lambda_s, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8) \quad (16)
\]

The contribution indices represent the change of generation and demand of any bus in a system. If we assume the sum of contribution indices of generation and demand sides is zero, it implies that the change of real power loss is compensated by a selected slack bus.

#### 3.2 Gradient Vector of the Increased Line Flow

Based on Fig.1, the real and reactive power flow at the receiving end bus can be expressed in (17) and (18) respectively.

\[
P_{ijr} = V_{ij}Y_{ij} \cos \theta_{ij} - V_{ij}Y_{ij} \cos (\theta_{ij} + \delta_i - \delta_j) \quad (17)
\]

\[
Q_{ijr} = V_{ij}Y_{ij} \sin \theta_{ij} - V_{ij}Y_{ij} \sin (\theta_{ij} - \delta_j) \quad (18)
\]

where the terms \( Y_{ij} \Delta \theta_{ij} \) represents elements of the bus admittance matrix.

The terms need to be calculated first are the real and reactive power sensitivity factors flowing into the receiving end bus compared to the real and reactive power injected into the generation bus and load bus, i.e., \( \partial P_{ijr}/\partial P_m, \partial Q_{ijr}/\partial P_m, \partial P_{ijr}/\partial Q_k \) and \( \partial Q_{ijr}/\partial Q_k. \) The subscripts in represents all buses except the slack bus, and \( k \) represents all load buses. All the sensitivity factors can be expressed by (19)-(22).

\[
\frac{\partial P_{ijr}}{\partial \delta_m} = \sum_{u, m \in G \cup L} \left[ \frac{\partial P_{ijr}}{\partial P_u} \frac{\partial P_u}{\partial \delta_m} \right] + \sum_{m \in G \cup L} \left[ \frac{\partial P_{ijr}}{\partial Q_u} \frac{\partial Q_u}{\partial \delta_m} \right] \quad (19)
\]

\[
\frac{\partial P_{ijr}}{\partial \delta_k} = \sum_{u \in G \cup L} \left[ \frac{\partial P_{ijr}}{\partial P_u} \frac{\partial P_u}{\partial \delta_k} \right] + \sum_{u \in G \cup L} \left[ \frac{\partial P_{ijr}}{\partial Q_u} \frac{\partial Q_u}{\partial \delta_k} \right] \quad (20)
\]

\[
\frac{\partial Q_{ijr}}{\partial \delta_k} = \sum_{u, m \in G \cup L} \left[ \frac{\partial Q_{ijr}}{\partial P_u} \frac{\partial P_u}{\partial \delta_m} \right] + \sum_{u \in G \cup L} \left[ \frac{\partial Q_{ijr}}{\partial Q_u} \frac{\partial Q_u}{\partial \delta_m} \right] \quad (21)
\]

\[
\frac{\partial Q_{ijr}}{\partial \delta_k} = \sum_{u, m \in G \cup L} \left[ \frac{\partial Q_{ijr}}{\partial P_u} \frac{\partial P_u}{\partial \delta_k} \right] + \sum_{u \in G \cup L} \left[ \frac{\partial Q_{ijr}}{\partial Q_u} \frac{\partial Q_u}{\partial \delta_k} \right] \quad (22)
\]
Equations (19) and (20) can be represented by (23).

\[
\begin{bmatrix}
\frac{\partial P_{ijr}}{\partial V_i} \\
\frac{\partial Q_{ijr}}{\partial V_i}
\end{bmatrix}
= [J]^T \begin{bmatrix}
\frac{\partial P_{ijr}}{\partial Q_i} \\
\frac{\partial Q_{ijr}}{\partial Q_i}
\end{bmatrix}
\tag{23}
\]

Equations (21) and (22) can be represented by (24).

\[
\begin{bmatrix}
\frac{\partial P_{ijr}}{\partial V_i} \\
\frac{\partial Q_{ijr}}{\partial V_i}
\end{bmatrix}
= [J]^T \begin{bmatrix}
\frac{\partial P_{ijr}}{\partial V_i} \\
\frac{\partial Q_{ijr}}{\partial V_i}
\end{bmatrix}
\tag{24}
\]

The terms \(\partial P_{ijr}/\partial \delta_i, \partial P_{ijr}/\partial V_k, \partial Q_{ijr}/\partial \delta_m, \) and \(\partial Q_{ijr}/\partial V_k\) are zero for every bus except bus \(-i\) and bus \(-j\), of which the values can be obtained from (25)-(30).

\[
\frac{\partial P_{ijr}}{\partial \delta_i} = -\partial P_{ijr} = V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_i + \delta_j) \quad \tag{25}
\]

\[
\frac{\partial P_{ijr}}{\partial \delta_j} = -V_j Y_{ij} \cos(\theta_{ij} + \delta_i + \delta_j) \quad \tag{26}
\]

\[
\frac{\partial Q_{ijr}}{\partial V_i} = 2 V_j V_{ij} \cos(\theta_{ij}) - V_j Y_{ij} \cos(\theta_{ij} + \delta_i + \delta_j) \quad \tag{27}
\]

\[
\frac{\partial Q_{ijr}}{\partial \delta_i} = V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_i + \delta_j) \quad \tag{28}
\]

\[
\frac{\partial Q_{ijr}}{\partial \delta_j} = V_j Y_{ij} \sin(\theta_{ij} + \delta_i + \delta_j) \quad \tag{29}
\]

\[
\frac{\partial Q_{ijr}}{\partial V_j} = V_i V_{ij} \sin(\theta_{ij} + \delta_i + \delta_j) - 2 V_j Y_{ij} \sin(\theta_{ij}) - Y_c \quad \tag{30}
\]

\(J\) is a conventional Jacobian matrix, which can be divided into \(J_1, J_2, J_3\) and \(J_4\). \(J_1\) is an \(m \times m\) matrix, of which the diagonal and off-diagonal elements are presented in (31) and (32).

\[
\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1,\neq i}^{n} V_i V_j Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j), \text{ for } \sum_{j=1,\neq i}^{n} \quad \tag{31}
\]

\[
\frac{\partial P_i}{\partial \delta_j} = -V_j Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j), \text{ for } \sum_{j=1,\neq i}^{n} \quad \tag{32}
\]

\(J_2\) is a \(m \times k\) matrix, of which the diagonal and off-diagonal elements are shown in (33) and (34) respectively.

\[
\frac{\partial P_i}{\partial V_i} = 2 V_i Y_{ii} \cos(\theta_{ii}) + \sum_{j=1,\neq i}^{n} V_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j), \text{ for } \sum_{j=1,\neq i}^{n} \quad \tag{33}
\]

\[
\frac{\partial P_i}{\partial V_j} = -V_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) \quad \tag{34}
\]

\(J_3\) is a \(k \times m\) matrix. The diagonal and off-diagonal elements are shown in (35) and (36) respectively.

\[
\frac{\partial Q_i}{\partial \delta_i} = \sum_{j=1,\neq i}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j), \text{ for } \sum_{j=1,\neq i}^{n} \quad \tag{35}
\]

\[
\frac{\partial Q_i}{\partial \delta_j} = -V_j Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j) \quad \tag{36}
\]

\(J_4\) is a \(k \times k\) matrix, of which the diagonal and off-diagonal elements are expressed by (37) and (38).

\[
\frac{\partial P_i}{\partial V_j} = 2 V_j Y_{ij} \sin(\theta_{ij}), \text{ for } \sum_{j=1,\neq i}^{n} \quad \tag{37}
\]

\[
\frac{\partial Q_i}{\partial V_j} = -V_j Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j) \quad \tag{38}
\]

Using (31)-(38), we can rewrite (23) and (24) into (39) and (40) respectively.

\[
\left[
\begin{bmatrix}
\frac{\partial P_{ijr}}{\partial \delta_i} \\
\frac{\partial Q_{ijr}}{\partial \delta_i}
\end{bmatrix}
\right] = \left[
\begin{bmatrix}
J_1 \\
J_2
\end{bmatrix}
\right]^{-1}
\begin{bmatrix}
\frac{\partial P_{ijr}}{\partial Q_i} \\
\frac{\partial Q_{ijr}}{\partial Q_i}
\end{bmatrix}
\tag{39}
\]

\[
\left[
\begin{bmatrix}
\frac{\partial P_{ijr}}{\partial \delta_i} \\
\frac{\partial Q_{ijr}}{\partial \delta_i}
\end{bmatrix}
\right] = \left[
\begin{bmatrix}
J_1 \\
J_2
\end{bmatrix}
\right]^{-1}
\begin{bmatrix}
\frac{\partial P_{ijr}}{\partial V_k} \\
\frac{\partial Q_{ijr}}{\partial V_k}
\end{bmatrix}
\tag{40}
\]

We can solve (39) and (40) to obtain \(\partial P_{ijr}/\partial P_m, \partial Q_{ijr}/\partial P_m, \partial P_{ijr}/\partial Q_k, \) and \(\partial Q_{ijr}/\partial Q_k\), which can be used to calculate the gradient vector of each receiving end line flow, denoted by \(\Delta P_{ijr} + j \Delta Q_{ijr}\), of which the real and imaginary parts can be expressed in (41) and (42).

\[
\Delta P_{ijr} = \sum_{m \in G \cup L} \left[\frac{\partial P_{ijr}}{\partial P_m} \cdot \Delta P_m \right] + \sum_{k \in L} \left[\frac{\partial P_{ijr}}{\partial Q_k} \cdot \Delta Q_k \right] \quad \tag{41}
\]

\[
\Delta Q_{ijr} = \sum_{m \in G \cup L} \left[\frac{\partial Q_{ijr}}{\partial P_m} \cdot \Delta P_m \right] + \sum_{k \in L} \left[\frac{\partial Q_{ijr}}{\partial Q_k} \cdot \Delta Q_k \right] \quad \tag{42}
\]

The terms \(\Delta P_m\) and \(\Delta Q_k\) in (41) and (42) are referred to an incremental change of the real and reactive power at buses \(m\) and \(k\) respectively. They can be calculated by (43) and (44).

\[
\Delta P_m = \lambda_m \cdot \Delta P, \text{ for } m \in G \cup L \quad \tag{43}
\]

\[
\Delta Q_k = \lambda_k \cdot \Delta Q, \text{ for } k \in L \quad \tag{44}
\]

Substitute \(\Delta P_m\) and \(\Delta Q_k\) from (43) and (44) into (41) and (42), then rearrange these equations to result in (45) and (46).
\[
\Delta P_{ijr} = \sum_{m \epsilon G \cup L} \left[ \frac{\partial P_{ijr}}{\partial P_m} \cdot \lambda_m \right] \Delta P_m + \sum_{k \epsilon L} \left[ \frac{\partial P_{ijr}}{\partial Q_k} \cdot \lambda_m \right] \Delta Q_L
\]
\[
\Delta Q_{ijr} = \sum_{m \epsilon G \cup L} \left[ \frac{\partial Q_{ijr}}{\partial P_m} \cdot \lambda_m \right] \Delta P_m + \sum_{k \epsilon L} \left[ \frac{\partial Q_{ijr}}{\partial Q_k} \cdot \lambda_m \right] \Delta Q_L
\]

It should be noted again that the gradient vector; \(\Delta P_{ijr} + j \Delta Q_{ijr}\), is calculated for each iteration, from which \(\Delta S_{predicted}\) in (16) can be obtained.

### 3.3 The Predicted Collapsing Point

The predicted collapsing point, \((P_{ijr}, Q_{ijr})\), is the intersection between the voltage stability curve, defined in (11), and the line extended from the gradient vector, which is expressed in (47).

\[
Q_{ijr} = \left( \frac{\Delta Q_{ijr}}{\Delta P_{ijr}} \right) P_{ijr} + \left( Q_{ijr} - \frac{\Delta Q_{ijr}}{\Delta P_{ijr}} P_{ijr} \right)
\]

The (47) can be simplified to (48).

\[
Q_{ijr} = M \cdot P_{ijr} + C
\]

where \(M = \frac{\Delta Q_{ijr}}{\Delta P_{ijr}}\) and \(C = Q_{ijr} - \frac{\Delta Q_{ijr}}{\Delta P_{ijr}} P_{ijr}\).

The calculation method to obtain the predicted collapsing point can be illustrated in Fig.2.

### 3.4 Transmission Line Performance Indices

The transmission line performance indices proposed in this paper i.e., line security index, line security margin, line severity index and line severity index, can be defined as below.

#### 3.4.1 The Line Security Margin

The line security margin is defined as the norm of the vector starting from the current operating point to the predicted collapsing point as illustrated in Fig.3. The line security margin can be stated as (51).

\[
LSM_{ij} = \left\| (P_{ijr}, Q_{ijr}) (P_{ijr}, Q_{ijr}) - (P_{ijr}, Q_{ijr}) \right\|
\]

#### 3.4.2 Line and switch models

The line security index is defined as the ratio of the line security margin to the apparent power of the predicted collapsing point, which can be expressed by (52).

\[
LSI_{ij} = \left( P_{ijr} Q_{ijr} Q_{ijr} \right) \left( P_{ijr} Q_{ijr} \right)
\]

#### 3.4.3 The Line Severity Index

The line severity index can be defined as ratio of the norm of the gradient vector to the line security margin as shown in (53).

\[
LSv_{ij} = \left\| (P_{ijr}, Q_{ijr}) (P_{ijr}, Q_{ijr}) - (P_{ijr}, Q_{ijr}) \right\|
\]

### 3.5 Flowchart

The overall calculation procedures presented in 3.1-3.4 can be summarized as follow. First, we calculate the base-case power flow. For a current operating point, generation and load scenarios must be defined beforehand before go to the next step. Then,
for a specified line, a P-Q curve, a gradient vector, the predicted collapsing point, and the transmission line performance indices can be calculated sequentially. A P-Q curve and gradient vector of each line can be conducted using (12) and (45) and (46) respectively. Then, a predicted collapsing point can be calculated as present in section 3.3. Finally, three transmission line performance indices can be obtained from (51)-(53). It should be noted that, these indices are calculated at the current operating point according to a defined scenario. To perform calculation at the further operating point, power flow solution must be solved and used as a base-case of the next current operating point. The process may be repeated until diverging of the power flow calculation is found. The calculation procedures presented above can be shown in Fig.4.

![Flowchart of the overall calculation procedures](image)

**Fig.4:** Flowchart of the overall calculation procedures

### 4. SIMULATION AND RESULTS

The proposed method is tested with a modified 6-bus system [24]. In this system, buses—1, 2 and 3 are generator buses, whereas buses—4, 5 and 6 are load buses. All the required bus and line data are available in the appendix.

The simulation starts from a base-case with 100% load. Then total generation and demand are increased proportionally until reaching the collapsing point. For simplicity, a repetitive power flow is adopted. Generation and transmission thermal limit are not considered. This is a traditional assumption to smooth the loading trajectories by neglecting the automatic actions of controllers and protective equipments. The obtained results at some loading points are presented. At the beginning, it found that the gradient vector and the proposed performance indices change continuously. However, at the verge to the collapsing point line No.5 connected between bus-2 and 4 hits the limit first and then returns to the lower portion of the P-V curve. This incident results in the fluctuation of gradient vector direction, which is calculated from sensitivity factors as described in section 3.2. The line security margin, security index and severity index defined by (51), (52) and (53) are shown in Tables 1, 2 and 3 respectively.

It is clear that bus—4 is the weakest bus in this study due to the reason that some lines, e.g. lines No. 2 and 5 hit voltage stability limit before the others. In addition, it can be seen from all indices. From Table 1, security margin of these two lines are drop close to zero, 0.03 and 0.13 MVA respectively. Similar results are also shown in Table 2 for line security index, which are 0.0002 and 0.0005. It can be found that line No.2 is more severe than line No.5 since the severity index of line No.2 is 10772.7 whereas line No.5 is 2513.8 as shown in Table 3. For the lines connected be-tween generator buses, i.e. lines No.1 and 4 are safe from hitting voltage stability limit especially in the light load condition. It should be noted here that, at the proximity to the collapsing point, direction of the gradient vector cannot be calculated precisely. Thus, most of the security margin for all the lines increase rapidly. Accordingly, the security margin in the direction of gradient vector alone is not suitable to evaluate the capability of line loading. However, this effect is less impact to line security and line severity indices.

### 5. CONCLUSIONS

Three performance indices of a transmission line i.e., line security margin, line security and line severity indices are defined in this paper. These indices are formulated based on a static voltage stability criteria defined on a transmission line P-Q and can be computed from a process of calculating the maximum loading capability of power system with slightly modification from the traditional methods. Additionally, the proposed method provides information to evaluate severity of a transmission system, which can be used for transmission planning or upgrading purpose.
### Table 1: The Line Security Margin

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Connecting bus</th>
<th>Line Security Margin (MVA) at% of base-case Loading</th>
</tr>
</thead>
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<td>2</td>
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### Table 2: The Line Security Index

<table>
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<tr>
<th>Line No.</th>
<th>Connecting bus</th>
<th>Line Security Index at% of base-case Loading</th>
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<tbody>
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</tr>
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### Table 3: The Line Severity Index

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<th>Line Severity Index at% of base-case Loading</th>
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### APPENDIX

#### Table A1: Bus Data of a Modified 6-Bus System

<table>
<thead>
<tr>
<th>Bus</th>
<th>Voltage (p.u.)</th>
<th>Pg (MW)</th>
<th>Qg (MVar)</th>
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</tr>
<tr>
<td>PV 2</td>
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<td>-</td>
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<tr>
<td>SLACK</td>
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<td>PQ 4</td>
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</tr>
<tr>
<td>PQ 5</td>
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<td>0</td>
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</table>

#### Table A2: Bus Data of a Modified 6-Bus System

<table>
<thead>
<tr>
<th>Bus</th>
<th>V (p.u.)</th>
<th>X (p.u.)</th>
<th>Y (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.02</td>
</tr>
</tbody>
</table>

### Fig. A1: Topology of a modified 6-bus system
Transmission Line Performance Indices Calculation Based on Voltage Stability Criterion

References


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