A Method of Determining the Secondary Exponential Smoothing Parameter Based on OWA

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ABSTRACT

To predict the stock closing price, the method of the secondary exponential smoothing model (SESM) is used to a trend prediction. In order to determine smoothing parameter, ordered weighted aggregation (OWA) is considered and optimal model of \( \alpha \) is proposed. To obtain the weights of OWA, historical error and prediction error of SESM are defined. Our method is applied to San Jing drugs stock closing price forecast.

Keywords: SESM, OWA, Historical error, Prediction error, Stock prediction

1. INTRODUCTION

In social economy activity, people must carry on the forecasting in order to make the correct decision-making. For example, making predictions of the stock closing price of certain companies for the future certain time, then we buy some stocks according to forecast result. The stock market gradually becomes an important and essential part of the negotiable securities industry. In the stock transaction data analysis and the forecast research, the stock forecasting has theory and application value. Exponential smoothing methods (ESM) are considered a collection of ad hoc techniques for extrapolating various types of unvaried time series. Nowadays, ESM have widely applications [1]-[11].

In this paper, the stock closing price forecast is considered on SESM. In next Section, SESM and its smoothing parameter \( \alpha \) are introduced. In Section 3, determining smoothing parameter \( \alpha \) based on OWA is proposed, and optimal model of \( \alpha \) is shown. To obtain the weights of OWA, historical error and prediction error of SESM are defined. In Section 4, example analysis is given. The conclusion is Section 5.

2. SESM AND ITS SMOOTHING PARAMETER \( \alpha \)

Simple exponential smoothing model (ESM) usually works best for series exhibiting no marked seasonality or trend. When a series does have a strong trend or cyclical Simple exponential smoothing model (ESM) usually works best for series exhibiting no marked seasonality or trend. When a series does have a strong trend or cyclical

SESM uses a weighted average of past and current values, adjusting weight on current values to account for the effects of swings in the data, such as seasonality. Using an \( \alpha \) term (between 0-1), you can adjust the sensitivity of the smoothing effects. SESM is often used on large scale statistical forecasting problems, because it is both robust and easy to apply. SESM is a popular scheme to produce a smoothed time series. Whereas in single moving averages the past observations are weighted equally, SESM assigns exponentially decreasing weights as the observation get older. In other words, recent observations are given relatively more weight in forecasting than the older observations. SESM is defined as follows.

Suppose the primary data \( Y = \{y_1, y_2, \ldots, y_n\} \) are the original time sequence. Let \( S^{(1)}_t \) be first smoothing value at the time \( t \), and \( S^{(2)}_t \) be secondary smoothing value at the time of \( t \). Formally, they are expressed as follows

\[
\begin{align*}
S^{(1)}_t &= \alpha Y_t + (1-\alpha)S^{(1)}_{t-1} \\
S^{(2)}_t &= \alpha S^{(1)}_t + (1-\alpha)S^{(2)}_{t-1} \\
S^{(1)}_1 &= S^{(2)}_1 = y_1
\end{align*}
\]

in which, \( \alpha \) is smoothing parameter. According to (1), we obtain

\[ F_{t+m} = a_t + b_t m \]  (2)

in which, \( F_{t+m} \) is prediction value at the time \( t+m \), \( m \) is called prediction steps, \( m = \{0, 1, 2, \ldots, M\} \), and

\[
\begin{align*}
a_t &= 2S^{(1)}_t - S^{(2)}_t \\
b_t &= \frac{\alpha}{(1-\alpha)}(S^{(1)}_t - S^{(2)}_t)
\end{align*}
\]

(1) and (2) are called SESM.
In SESM, smoothing parameter α is important. For practice problem, is needed to be fixed. From this point of view, whether SESM is suitable for the problem is decided by α. The parameter α denotes the proportion of the new prediction data and the original prediction data, which is to say denotes the revise extent. So, α embodies the revise error of the prediction model. As we know, deciding α has two kinds of methods.

- Automatically assignation. This method is according to smallest sum square forecast error. Hence, self learning is property of the method.
- Artificial determinations. This method is according to observing \( Y = \{y_1, y_2, \ldots, y_n\} \). Generally, the method depends on human’s experience.

3. DETERMINING SMOOTHING PARAMETER α BASED ON OWA

In real world practice, aggregation operator is used if some system must make a judgment or synthesize certain related knowledge [12]. OWA is aggregation operator which aggregates some information with weight in a certain order, it is formalized as follows.

**Definition 1.** [13] A mapping \( f_{owa} \) from \( I^n \to I \) (where \( I = [0,1] \)) is called an OWA operator of dimension \( n \) if associated with \( f_{owa} \), is a weight vector \( W = [w_1, w_2, \ldots, w_n]^T \), such that \( w_i \in [0,1] \) and \( \sum w_i = 1 \), and \( f_{owa}(a_1, a_2, \ldots, a_n) = W^T \sigma(a) = w_1 a_{\sigma(1)} + w_2 a_{\sigma(2)} + \ldots + w_n a_{\sigma(n)} \) Where \( \sigma(1), \ldots, \sigma(n) \) is a permutation of \( \{1, \ldots, n\} \) such that \( a_{\sigma(i-1)} \geq a_{\sigma(i)} \) for all \( i = 2, \ldots, n \), i.e., \( a_{\sigma(i)} \) is the \( i \)th largest element in the collection \( a_1, a_2, \ldots, a_n \). \( \sigma \) is called ordered argument vector of \( (a_1, a_2, \ldots, a_n) \).

Let system \( y = f(x_1, \ldots, x_n) \), if α is fixed, then SESM \( S_\alpha \) of the system is also fixed, i.e., \( S_\alpha \) determines \( y = f_\alpha(x_1, \ldots, x_n) \), generally, let system error

\[
E_\alpha = \int_{\Omega} |f(x_1, \ldots, x_n) - f_\alpha(x_1, \ldots, x_n)| \, dx \tag{4}
\]

In which, \( \Omega \) is domain of the system. \( X = (x_1, \ldots, x_n) \). Selecting depends on \( E_\alpha \), i.e., the smaller \( E_\alpha \), the better α is. This process can be formalized by optimal model as follows

**Object:** \( \min E_\alpha \)

**S.t.:** \( \alpha \in [0,1] \) \tag{5}

However, in stock closing price system, \( f(x_1, \ldots, x_n) \) is unknown, only discrete stock closing price \( \{y_1, y_2, \ldots, y_n\} \) is known, hence, it is difficult to obtain \( E_\alpha \) for fixed α. The problem is converted as follows

1. For fixed \( E_\alpha \), let forecast closing price \( \{y_1^\alpha, y_2^\alpha, \ldots, y_n^\alpha\} \); 2. Mean error \( ME_\alpha = \sum_{i=1}^n e_i^\alpha \); in which, \( e_i = y_i^\alpha - y_i \);

3. Mean square error \( MSE_\alpha = \sum_{i=1}^n (e_i^\alpha)^2 \); 4. Mean absoluteness percent error \( MAPE_\alpha = \left( \frac{\sum_{i=1}^n |e_i^\alpha|}{n} \right) \times 100; \)

5. Historical error (HE) \( HE_\alpha = \frac{ME_\alpha + MSE_\alpha + MAPE_\alpha}{3} \);

6. Prediction error (PE) \( PE_\alpha = \sum_{m=1}^M (y_n+m - y_{n+m}^\alpha) \), in which, \( y_{n+m} \) is actual stock closing price at the time \( n + m \) and \( y_{n+m}^\alpha \) is prediction stock closing price at the time \( n + m \).

7. System error \( \alpha \) is a function decided by user.

Based on above expressions, (5) is modified as follows

**Object:** \( \min g(HE_\alpha, PE_\alpha) \)

**S.t.:** \( \alpha \in [0,1] \) \tag{6}

Obviously, optimal value depends on \( g \) in (6). How to select \( g \) and solving (6) is a problem. In this paper, the following method is adopted.

From real world practice point of view, the smaller \( HE_\alpha \) and \( PE_\alpha \) are, the better \( \alpha \) is. Formally, random selection \( \{\alpha_1, \ldots, \alpha_n\} \), is reduced from \( \{\alpha_1, \ldots, \alpha_n\} \) if there exists \( \alpha_j \) such that \( HE_{\alpha_j} < HE_\alpha \) and \( PE_{\alpha_j} < PE_\alpha \). Let reduced set be \( \{\alpha_1, \ldots, \alpha_K\} \), \( K \leq n \) then \( \alpha_{new} \) need to be found according to \( \{\alpha_1, \ldots, \alpha_K\} \). In this paper, \( \alpha_{new} \) is determined as follows.

\[
\alpha_{new} = f_{owa}(\alpha_1, \ldots, \alpha_K) = \sum_{k=1}^K w_k \alpha_k \tag{7}
\]

In which, \( w_k \) is decided as follows.

\[
\hat{w}_k = g(HE_{\hat{\alpha}_k}, PE_{\hat{\alpha}_k}) \tag{8}
\]

\[
\hat{w}_k = \frac{\hat{w}_k}{\sum_{i=1}^K \hat{\alpha}_k} \tag{9}
\]

In this paper, \( g \) is selected as follows

\[
\begin{cases}
 g(HE_{\hat{\alpha}_k}, PE_{\hat{\alpha}_k}) = p_1 HE_{\hat{\alpha}_k} + p_2 PE_{\hat{\alpha}_k} \\
p_1 + p_2 = 1
\end{cases} \tag{10}
\]

in which, \( p_1 \) and \( p_2 \) are weights of \( HE_{\hat{\alpha}_k} \) and \( PE_{\hat{\alpha}_k} \), respectively

4. EXAMPLE ANALYSIS

We make short-term closing price forecast of San Jing drugs stock according to the 16 transaction data (from March 3, 2006 to March 24, 2006, see Table1) [14].

Let \( \alpha_1 = 0.3, \alpha_2 = 0.5, \alpha_3 = 0.7 \), According to (2) and (3), we have \( a_{16} = 7.2319, b_{16} = 0.2759; F_{17} = a_{16} + b_{16} = 7.5078; F_{18} = a_{16} + b_{16} = 7.7837; F_{19} = a_{16} + b_{16} = 8.0596 \). The analysis of the error for 3-16 transaction date closing price.
Table 1: $\alpha = 0.5$ Closing price table

<table>
<thead>
<tr>
<th>Date</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$s_1^{[1]}$</th>
<th>$s_1^{[2]}$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$P_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>6.20</td>
<td>6.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>6.35</td>
<td>6.295</td>
<td>0.2125</td>
<td>0.2175</td>
<td>0.0755</td>
<td>0.0705</td>
<td></td>
</tr>
<tr>
<td>3.7</td>
<td>6.45</td>
<td>6.375</td>
<td>0.17</td>
<td>0.19</td>
<td>0.06</td>
<td>0.055</td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td>6.51</td>
<td>6.418</td>
<td>0.0944</td>
<td>0.1042</td>
<td>0.05</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>6.52</td>
<td>6.574</td>
<td>0.0314</td>
<td>0.0364</td>
<td>0.05</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>3.9</td>
<td>6.54</td>
<td>6.612</td>
<td>0.0402</td>
<td>0.0492</td>
<td>0.05</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>3.14</td>
<td>6.60</td>
<td>5.979</td>
<td>0.3988</td>
<td>0.3970</td>
<td>0.0055</td>
<td>0.0052</td>
<td></td>
</tr>
<tr>
<td>3.13</td>
<td>6.60</td>
<td>6.036</td>
<td>0.3962</td>
<td>0.3954</td>
<td>0.0045</td>
<td>0.0042</td>
<td></td>
</tr>
<tr>
<td>3.10</td>
<td>6.57</td>
<td>5.998</td>
<td>0.3985</td>
<td>0.3971</td>
<td>0.0027</td>
<td>0.0022</td>
<td></td>
</tr>
<tr>
<td>3.17</td>
<td>6.68</td>
<td>6.039</td>
<td>0.0062</td>
<td>0.0056</td>
<td>0.0037</td>
<td>0.0032</td>
<td></td>
</tr>
<tr>
<td>3.20</td>
<td>6.20</td>
<td>6.150</td>
<td>0.0060</td>
<td>0.0054</td>
<td>0.0028</td>
<td>0.0024</td>
<td></td>
</tr>
<tr>
<td>3.21</td>
<td>6.10</td>
<td>6.175</td>
<td>0.0061</td>
<td>0.0056</td>
<td>0.0037</td>
<td>0.0032</td>
<td></td>
</tr>
<tr>
<td>3.22</td>
<td>6.45</td>
<td>6.394</td>
<td>0.0061</td>
<td>0.0056</td>
<td>0.0028</td>
<td>0.0024</td>
<td></td>
</tr>
<tr>
<td>3.23</td>
<td>6.63</td>
<td>6.268</td>
<td>0.0042</td>
<td>0.0037</td>
<td>0.0027</td>
<td>0.0024</td>
<td></td>
</tr>
<tr>
<td>3.24</td>
<td>6.40</td>
<td>6.360</td>
<td>0.0041</td>
<td>0.0036</td>
<td>0.0026</td>
<td>0.0023</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Error analysis for different $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\text{MS}$</th>
<th>$\text{MSE}$</th>
<th>$\text{MAPE}$</th>
<th>$\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.3$</td>
<td>0.0889</td>
<td>0.0165</td>
<td>0.49</td>
<td>0.0831</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
<td>0.0664</td>
<td>0.0706</td>
<td>2.1190</td>
<td>0.2478</td>
</tr>
<tr>
<td>$\alpha = 0.7$</td>
<td>0.075</td>
<td>0.1085</td>
<td>4.74</td>
<td>0.3948</td>
</tr>
</tbody>
</table>

1. if $p_1 = p_2 = 0.5$, then $\alpha_{new} = 0.37$
2. if $p_1 = 0.8, p_2 = 0.2$, then $\alpha_{new} = 0.39$
3. if $p_1 = 0.2, p_2 = 0.8$, then $\alpha_{new} = 0.34$

Finally, $\alpha_{new} = 0.34$ is the optimal solution of SESM (see fig2).

Fig.1: Graphical comparison between prediction curves and actual curves

Fig.2: Concatenated decoder.

5. CONCLUSIONS

In this paper, SESM is used to predict the stock closing price. In order to determine smoothing parameter, an optimal model of is proposed, and OWA is considered to obtain optimal solution. To obtain the weights of OWA, historical error and prediction error of SESM are defined. Our method is applied to
San Jing drugs stock closing price forecast.

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References


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