FFT/IFFT Based Blind SIMO Channel Identification

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ABSTRACT

This paper presents an FFT/IFFT based blind identification method for estimating the finite impulse response of single-input multiple-output channels driven by an unknown deterministic signal. The proposed algorithm successfully handles a very small size of received data, for which the existing blind channel estimation methods, including the subspace, cross-relation and shifted correlation algorithms, are known to be ineffective. Moreover, with no assumption of the precise knowledge of channel order, the proposed algorithm is capable of estimating channel parameters as well as detecting channel order. Simulations show that the proposed algorithm outperforms the existing methods in small sample size situations.

1. INTRODUCTION

Single-input multiple-output (SIMO) channels arise in wireless communications from radio propagation between a single transmitter (e.g., base station) and multiple receivers/sensors. SIMO channels can also be used to model the effect of oversampling the received signal of a data communication system [1] in the single sensor case. When the source signal is unknown, channel estimation is “blind”, that is, channel impulse response is estimated using only channel outputs. Blind channel identification techniques have developed rapidly since early 1990s; see [7], [8], [2]. Among those important techniques are deterministic methods, e.g., the cross-relation (CR) algorithm [4], and second-order statistics based methods, e.g., the subspace (SS) algorithm [3], and more recently, the shifted correlation (SC) algorithm [5] [6]. The SS and CR methods have the desirable feature of finite sample convergence (FSC), i.e., exact channel identification is achieved with a finite sample size in the absence of noise. However, both SS and CR methods require the knowledge of channel order to effect channel estimation. Although the SC algorithm [5] [6], which is based on shifted correlation matrices of outputs, does not have the FSC property, it assumes no knowledge of the channel order and achieves channel parameter identification and order (over)estimation. Despite different individual strengths, these methods share a common deficiency - they become ineffective when the observation data size is very small. This issue needs addressing because in practical communication applications, there exist situations where a long data sequence is unavailable.

In this paper, we present a novel blind SIMO channel identification method targeted at a small size of observation data. Motivated by the CR between each channel output pair, which serves as the basis of the CR approach [4], we extend the CR property to the frequency domain via the discrete Fourier transform (DFT) and take advantage of the computational efficiency of the fast Fourier transform (FFT). The new method makes two contributions to blind channel identification. First, with a null guard interval introduced in the input, the proposed algorithm requires less observation data than the existing methods, and a single short-duration output block suffices for channel identification. Second, assuming only the knowledge of the upper bound of channel order, the proposed algorithm is capable of estimating channel parameters as well as detecting channel order. In the case when channel order is unknown, compared to the SC approach [5] [6], our method performs better in small sample size scenarios. With FSC, the proposed algorithm provides a substantial performance improvement over the SC approach in the high SNR region.

The paper is organized as follows. Section II presents the SIMO channel model. Section III develops the new algorithm for SIMO channel identification and order detection. Simulation results are shown in Section IV. Conclusion is given in Section V.

2. SIMO CHANNEL MODEL

Let us consider the following discrete finite impulse response (FIR) $L$-channel model:

$$x_m(n) = s(n) * h_m(n) + w_m(n), \quad m = 1, 2, \cdots, L \quad (1)$$

where $x_m(n) \in \mathbb{C}$ is the $m$th channel output at time $n$ with $\mathbb{C}$ denoting the set of complex numbers, $s(n) \in \mathbb{C}$ the common input, $h_m(n) \in \mathbb{C}$ the impulse response of the channel $m$, and $w_m(n) \in \mathbb{C}$ the additive noise at channel $m$ (uncorrelated with the source signal). The symbol $*$ in (1) denotes convolution.
The maximum order of the \( L \) channels is \( M \), where \( M \) is not necessarily known a priori, but upper bounded by a known integer \( M \), i.e., \( M \leq M \). Without loss of generality, there exists a channel \( q \), \( 1 \leq q \leq L \), such that \( h_q(0) \), \( h_q(M) \neq 0 \).

The data structure used in our problem setting is now explained. Suppose that an observation data block is of length \( N \). The input sequence \( s(n) \) can be transmitted through multiple blocks of fixed length or a single block. The transmission block is made up of source symbols of size \( N - M \) followed by \( M \) zeros. A received data block is expressed by

\[
x_m = \mathcal{H}_m s + w_m, \quad m = 1, 2, \ldots, L
\]

where \( x_m = [x_m(0) \ x_m(1) \ \cdots \ x_m(N - 1)]^T \),

\[
s = [s(0) \ s(1) \ \cdots \ s(N - M - 1)]^T,
\]

\[
w_m = [w_m(0) \ w_m(1) \ \cdots \ w_m(N - 1)]^T,
\]

and \( \mathcal{H}_m \in \mathbb{C}^{N \times (N - M)} \) is a Toeplitz matrix whose first column is \([h_m(0) \ h_m(1) \ \cdots \ h_m(M - 1)]^T\) and first row is \([h_m(0) \ 0 \ \cdots \ 0]^T\). The appended zeros in the input sequence \( s(n) \) are accounted for in the structure of \( \mathcal{H}_m \).

Appending a sequence of zeros to a data block, known as a guard interval, has been used in blind identification of single-input single-output systems [10]. Despite consuming bandwidth, a guard interval offers many advantages [10]. We exploit this idea as a way of clearing channel memory, and more importantly, reducing the amount of output data required for channel identification. As will be shown, with the SIMO model (2), using a single received block \( x_m \) of size \( N_s \), where \( N_s \) is small (as long as \( N_s \geq M + 1 \)), channel impulse response can be perfectly retrieved in the absence of noise.

3. FFT/IFFT BASED BLIND CHANNEL IDENTIFICATION

3.1 Theoretical Analysis

For convenience of presentation and derivation, we assume for now that channel order \( M \) is known. This assumption will later be replaced by only the knowledge of the upper bound \( M \).

For each noiseless output pair, the following CR holds:

\[
x_i(n)h_j(n) = x_j(n)h_i(n), \quad 1 \leq i, j \leq L, \ i \neq j \quad (3)
\]

Taking the \( N \)-point DFT on both sides of (3), where \( N \geq N_s + M \), we have

\[
X_i(k)H_j(k) = X_j(k)H_i(k), \quad k = 0, 1, \ldots, N - 1 \quad (4)
\]

where \( X_m(k) \) and \( H_m(k) \) represent the frequency-domain samples of \( x_m(n) \) and \( h_m(n) \), respectively, with \( m = i, j \). For each \( k = 0, 1, \ldots, N - 1 \),

\[
x_{i,j}(k) = [0 \ \cdots \ 0 -X_{j}(k) \ 0 \ \cdots \ 0 X_{i}(k) \ 0 \ \cdots \ 0]
\]

where the \( i \)th entry is \(-X_{j}(k)\) and the \( j \)th entry is \( X_{i}(k)\). For each \( k = 0, 1, \ldots, N - 1 \), we can form

\[
X_kH_k = 0 \quad (6)
\]

where \( H_k = [H_1(k) \ H_2(k) \ \cdots \ H_L(k)]^T \in \mathbb{C}^L \), and

\[
X_k = [X_1(k)^T \ X_2(k)^T \ \cdots \ X_L(k)^T] \in \mathbb{C}^L
\]

The following lemma specifies conditions upon the channels and the source signal to ensure identifiability.

**Lemma 1:** If there is no common zero among all the channels and the \( N \)-point DFT \( S(k) \) of the input sequence \( s(n) \) are nonzero, then any nontrivial solution \( \hat{H}_k \) to \( X_kH_k = 0 \), \( k = 0, 1, \ldots, N - 1 \), is given by \( \hat{H}_k = \alpha_kH_k \), where \( \alpha_k \) is a nonzero scalar dependent on \( k \).

**Proof:** \( X_{i,j}(k) \) in (5) is also expressed by

\[
X_{i,j}(k) = (S(k)[0 \ \cdots \ 0 -H_{j}(k) \ 0 \ \cdots \ 0 H_{i}(k) \ 0 \ \cdots \ 0]) = S(k)H_{i,j}(k)
\]

which leads to \( X_k = S(k)H(k) \), where

\[
S(k) = \text{diag}\{S(k), \ldots, S(k)\} \in \mathbb{C}^{L(N-\infty) \times L(N-\infty)},
\]

\[
H(k) = [H_{1,2}(k)^T \ H_{1,3}(k)^T \ \cdots \ H_{1,L}(k)^T \ H_{1,2}(k)^T \ \cdots \ H_{2,L}(k)^T \ \cdots \ H_{L-1,L}(k)^T]^T \in \mathbb{C}^{L(N-\infty) \times L}
\]

Hence, \( X_k\hat{H}_k = 0 \) is equivalent to \( S(k)H(k)\hat{H}_k = 0 \). As \( S(k) \neq 0 \), \( S(k) \) is nonsingular, and \( H(k)\hat{H}_k = 0 \). That is,

\[
H_{i}(k)\hat{H}_{j}(k) = H_{j}(k)\hat{H}_{i}(k), \text{ for any } 1 \leq i, j \leq L, \ i \neq j \quad (7)
\]

For \( k = 0, 1, \ldots, N - 1 \), there exists at least one channel \( l \) such that \( H_{l}(k) \neq 0 \). Since if for each \( m = 1, \ldots, L, H_m(k) = 0 \), based on the equivalence between the \( z \)-transform and DFT, \( H_m(z)|_{z=e^{j2\pi k/N}} = 0 \), thus contradicting the fact of no common zero among all channels. When \( H_i(k) \neq 0 \), we need to show that \( \hat{H}_i(k) \neq 0 \). We proceed with the proof by contradiction. Assume \( \hat{H}_i(k) = 0 \). According to (7),

\[
H_i(k)\hat{H}_i(k) = H_j(k)\hat{H}_i(k), \text{ for any } 1 \leq i, j \leq L, \ i \neq 1 \quad (8)
\]

If \( H_i(k) \neq 0 \) and \( \hat{H}_i(k) = 0 \), then \( \hat{H}_i(k) = 0 \). This means \( H_m(k) = 0 \) for each \( m = 1, \ldots, L, \) which contradicts that \( \hat{H}_k \) is a nontrivial solution to \( X_k\hat{H}_k = 0 \).
Thus, $\tilde{H}(k) \neq 0$ when $H_l(k) \neq 0$, or equivalently, $H_l(k) = \alpha_k$, where $\alpha_k$ is a nonzero scalar dependent on $k$. It follows from (8) that $H_l(k) = \alpha_k \tilde{H}(k)$. That is, $H_m(k) = \alpha_k \tilde{H}(k)$, $m = 1, \ldots, L$.

Based on Lemma 1, we can find $\tilde{H}(m)$, $m = 1, \ldots, L$. But we also need to find $\alpha_k$ so that taking the $N$-point IDFT on $H_m(k) = \alpha_k \tilde{H}(k)$ yields the channel impulse response $h_m(n)$. Expanding the IDFT of $H_m(k)$ gives

$$h_m(n) = \sum_{k=0}^{N-1} H_m(k) V^{-kn} = \sum_{k=0}^{N-1} \alpha_k \tilde{H}(k) V^{-kn}$$

(9)

where $V = e^{-j2\pi/N}$ and the scaling factor $1/N$ is omitted. Incorporating all channels, we rewrite (9) as

$$\begin{bmatrix}
H_{1,1} & H_{1,2} & \cdots & H_{1,M+1} \\
H_{2,1} & H_{2,2} & \cdots & H_{2,M+1} \\
\vdots & \vdots & \ddots & \vdots \\
H_{L,1} & H_{L,2} & \cdots & H_{L,M+1}
\end{bmatrix}
=\begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\vdots \\
\alpha_{N-1}
\end{bmatrix}
\begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_L
\end{bmatrix}$$

(10)

where $h_m = \{h_m(0), h_m(1), \ldots, h_m(M)\}^T$, and $H_{m,1} = \alpha_0 \tilde{H}(n) = \{h_m(0), h_m(1), \ldots, h_m(M)\}^T \in \mathbb{C}^{M+1 \times N}$. The top $M + 1$ rows (bottom $N - M - 1$ rows) of the matrix

$$V \cdot \text{diag}\{H_{m}(0), H_{m}(1), \ldots, H_{m}(N-1)\}$$

The elements of the IDFT matrix $V \in \mathbb{C}^{N \times N}$ are $V^{ij} = V^{-ij}$, for $i = 0, 1, \ldots, N-1$ and $j = 0, 1, \ldots, N-1$.

It is seen from (10) that the vector $[\alpha_0, \alpha_1, \ldots, \alpha_{N-1}]^T$ is in the null space of

$$\mathcal{H}_{N-M-1} = \begin{bmatrix}
H_{1,1} & H_{1,2} & \cdots & H_{1,M+1} \\
H_{2,1} & H_{2,2} & \cdots & H_{2,M+1} \\
\vdots & \vdots & \ddots & \vdots \\
H_{L,1} & H_{L,2} & \cdots & H_{L,M+1}
\end{bmatrix}
\in \mathbb{C}^L(M+\infty) \times N$$

In order to uniquely (up to a constant factor) determine the channel vector $h = [h_0^T, h_1^T, \ldots, h_L^T]^T \in \mathbb{C}^{L(M+\infty)}$, the nullity of $\mathcal{H}_{N-M-1}$ has to be one. As the dimension of $\mathcal{H}_{N-M-1}$ is $[L(N-M-1)] \times N$, to render nullity($\mathcal{H}_{N-M-1}$) = 1, we must have $L(N-M-1) \geq N-1$, i.e., $N \geq \frac{L(M+1)}{L-1}$, which is guaranteed by choosing the DFT size $N \geq N_s + M$, where $N_s \geq M + 1$.

**Lemma 2:** The nullity of $\mathcal{H}_{N-M-1}$ is one.

**Proof:** First we prove that the nullity of $\mathcal{H}_{N-M-1}$ is one, where $\mathcal{H}_{N-M-1}$ is in the same format as $\mathcal{H}_{N-M-1}$ except to replace $\tilde{H}(k)$ with $H_m(k)$. It is known that the set $[1 V V^2i \cdots V^{(N-1)i}]^T$ is a basis for $\mathbb{C}^N$ and that the first vector $[1 1 \cdots 1]^T$ for $i = 0$ of this set is in the null space of $\mathcal{H}_{N-M-1}$. Now we need to show that $v_i = [1 V V^2i \cdots V^{(N-1)i}]^T$, $i = 1, 2, \ldots, N-1$, and their linear combination are not in the null space of $\mathcal{H}_{N-M-1}$.

As $\mathcal{H}_{N-M-1} = [v_1 v_2 \cdots v_{N-1}] = [T_1^T T_2^T \cdots T_{N-1}^T]^T$, where $T_m \in \mathbb{C}^{N-M-\infty} \times (N-\infty)$ is a Toeplitz matrix whose first row is $[h_0(M) h_m(M-1) \cdots h_m(0) \cdots 0]$ and first column is $[h_0(M) 0 \cdots 0]^T$, it follows that each column of $[T_1^T T_2^T \cdots T_{N-1}^T]^T$ always contains the nonzero element $h_q(0)$ or $h_q(M)$ for some channel $q$, where $1 \leq q \leq L$. So $v_1$, $v_2$, $\ldots$, $v_{N-1}$ are not in the null space of $\mathcal{H}_{N-M-1}$.

Next we shall prove that any linear combination of $v_1$, $v_2$, $\ldots$, $v_{N-1}$ is not in the null space of $\mathcal{H}_{N-M-1}$. We proceed by contradiction. Suppose that there exist coefficients $a_1, a_2, \ldots, a_{N-1}$, not all zero, such that $\mathcal{H}_{N-M-1} \cdot [a_1 v_1 + a_2 v_2 + \cdots + a_{N-1} v_{N-1}] = 0$, i.e., $[T_1^T T_2^T \cdots T_{N-1}^T]^T [a_1 a_2 a_{N-1}] = 0$. Expanding this equation yields $h_m(n) \ast a_0 = 0$, where $\ast$ denotes linear convolution whose available length is $N-M-1$, $m = 1, \ldots, L$, and $\{a_n\}_{n=1}^{N-1}$ behaves as a common input sequence to the channels. Since this linearly convolved signal is identical to the corresponding portion of the $(N-1)$-point circular convolution, taking the $(N-1)$-point DFT gives $H_m(k)A(k) = 0$. As the DFT $A(k)$ of the input $a_n$ are required to be nonzero, $H_m(k) = 0$ for $m = 1, \ldots, L$, which means $H_m(z)|_{z=\exp(2\pi i/N)} = 0$, thus contradicting that all channels share no common zero.

We have shown nullity($\mathcal{H}_{N-M-1}$) = 1. Since $\mathcal{H}_{N-M-1} = \mathcal{H}_{N-M-1} \cdot \text{diag}\{1/\alpha_0, 1/\alpha_1, \ldots, 1/\alpha_{N-1}\}$, rank($\mathcal{H}_{N-M-1}$) = rank($\mathcal{H}_{N-M-1}$). Hence, nullity($\mathcal{H}_{N-M-1}$) = 1.

**Remark:** Determination of channel order is hinted at by the structure of the matrix $\mathcal{H}_{N-M-1}$. We can easily verify that for $M' > M$ or $M' < M$, nullity($\mathcal{H}_{N-M-1}$) $\neq 1$. Thus, lemma 2 offers an approach to order detection.

### 3.2 Algorithm Description

The theoretical development in the previous section has provided a solution to exact channel identification in the absence of noise. Hereafter we drop the assumption of the knowledge of channel order and assume only the knowledge of its upper bound $M$. This affects the size $N$ of the DFT taken on the observation data, namely, we require that $N \geq N_s + M$. In the presence of noise, the blind identification problem is solved in the least squares sense. A computational procedure of the proposed algorithm is illustrated as follows.

**Step 1** Choose $N \geq N_s + M$ and take the $N$-point FFT on each channel output vector $x_m$ and form the matrix $X_k$ in (6).

**Step 2** For $k = 0, 1, \ldots, N-1$, find $H_k$ that minimizes $\|X_k H_k\|_2$ subject to some nontrivial con-
Table 1: Finite impulse response for a discrete 4-channel system

<table>
<thead>
<tr>
<th>n</th>
<th>$h_1(n)$</th>
<th>$h_2(n)$</th>
<th>$h_3(n)$</th>
<th>$h_4(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.049+0.359i</td>
<td>0.443-0.0634i</td>
<td>-0.211-0.322i</td>
<td>0.417+0.030i</td>
</tr>
<tr>
<td>1</td>
<td>0.482-0.569i</td>
<td>1.0</td>
<td>-0.199+0.918i</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>-0.556+0.587i</td>
<td>0.921-0.194i</td>
<td>1.0</td>
<td>0.873+0.145i</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>0.189-0.208i</td>
<td>-0.284-0.524i</td>
<td>0.285+0.309i</td>
</tr>
<tr>
<td>4</td>
<td>-0.171+0.061i</td>
<td>-0.087-0.054i</td>
<td>0.136-0.19i</td>
<td>-0.049+0.161i</td>
</tr>
</tbody>
</table>

Step 3 For $l = 0, 1, \ldots, \min\{N_s-1, M\}$, compute the ratio of the second smallest singular value to the smallest singular value of $\tilde{\mathbf{H}}_{N-l-1}$. The value of $l$ corresponding to the largest ratio is the channel order $M$.

Step 4 Determine the vector $\mathbf{\alpha} = [\alpha_0 \alpha_1 \cdots \alpha_{N_s-1}]^T$ that minimizes $\|\tilde{\mathbf{H}}_{N-M-1}\mathbf{\alpha}\|_2$ subject to some non-trivial constraint, e.g., $\|\mathbf{\alpha}\|_2 = 1$.

Step 5 With $\mathbf{H}_k = [\tilde{H}_1(k) \tilde{H}_2(k) \cdots \tilde{H}_M(k)]^T$ obtained in Step 2, for $k = 0, 1, \ldots, N-1$ and $m = 1, \ldots, L$, compute $\alpha_k \tilde{H}_m(k)$. For each $m = 1, \ldots, L$, take the $N$-point IFFT on $\alpha_k \tilde{H}_m(k)$. The first $M+1$ elements of the IFFT are the FIR estimate for channel $m$.

Based on the theoretical development above, we make the following comments:

i. The presented method possesses the FSC (finite sample convergence) property. Due to FSC, the new algorithm converges quickly as the SNR tends to $\infty$.

ii. The identifiability studied in the paper belongs to CR-based identifiability, which is one category of identifiability defined in [9]. Equivalence between several identifiability notions was established in [9], which showed that observation duration has to be greater than $3M$ to ensure identifiability in whatever sense. By contrast, the proposed algorithm requires observation data of length $N_s \geq M+1$ to estimate the channels.

iii. The main computational load of the proposed algorithm comes from singular value decomposition (SVD). Specifically, when channel order is known, SVD is taken $N$ times on the $L(N-1) \times L$ matrix $X_k$ and once on the $L(N-M-1) \times N$ matrix $\tilde{\mathbf{H}}_{N-M-1}$, where $N$ is the FFT size and $N \geq N_s + M$. As far as computational cost is concerned, first, the proposed algorithm is suitable for a small output sample size $N_s$, and second, a tradeoff is necessary in terms of SVD operations and FFT computations. When $N_s$ is large, the proposed algorithm may not be computationally efficient compared to the SS [3], and SC [5] [6] methods.

iv. The presented method achieves both channel order detection and parameter estimation, which is a clear advantage over many existing methods that require the knowledge of channel order. Moreover, according to the computational procedure described above, it is straightforward, in principle, to determine channel order.

4. SIMULATIONS

Simulations were conducted to compare the proposed algorithm with the SS [3], CR [4] and SC [5] [6] methods. We used the SIMO channel example in [3] with channel coefficients shown in Table 1.

The SIMO channel was driven by a white binary (1 or -1) process of unit variance with additive white Gaussian noise applied to channel outputs. We computed the mean-square-error (MSE) to be the performance measure:

$$\text{MSE(dB)} = 10 \log_{10} \left( \frac{1}{t} \sum_{i=1}^{t} \|\hat{h}_i - h\|^2 \right)$$

where $t$ is the number of Monte Carlo runs ($t = 100$ was used), $h$ is the true (unit-norm) channel vector, and $\hat{h}_i$ is the estimated channel vector (with unit norm) from the $i$th run.

![Fig.1: Performance comparison of the SS, CR, SC, and proposed algorithms. Exact channel order known.](image-url)
sample size cannot be handled by the SS, CR and SC methods, for which ten times more (50) output samples were used. In this simulation, channel order was assumed to be known. The FFT size \( N = 16 \) was chosen for the proposed algorithm. The smoothing factor [6] was chosen to be 10 and equalization peak criterion (EPC) was applied to the SC approach. Fig. 1 compares the performance of different methods, which shows that the proposed algorithm performs satisfactorily for a very small data size. As indicated in [6], the SC algorithm levels off once SNR has reached a certain value. This is also observed in Fig. 1.

![Fig. 1: Comparison of the SC algorithm and the proposed algorithm. Channel order unknown.](image)

In the second simulation, we performed channel identification and order detection. The channel order was assumed unknown with an upper bound \( M = 10 \). As the SS [3] and CR [4] algorithms require the knowledge of channel order (and hence cannot operate under this condition), we only compared the performance of the SC method [5] [6] and the proposed algorithm. The simulation result for a block of 30 output samples is reported in Fig. 2. The FFT size \( N = 40 \). Again smoothing factor [6] equal to 10 and EPC were used for the SC algorithm. It is shown that for a comparatively small sample size, the proposed algorithm outperforms the SC method over a range of SNR. In particular, the higher the SNR, the more performance improvement of the proposed algorithm over the SC method. This is due to the FSC property of the proposed algorithm and the flooring characteristic of the SC approach. It is worth noting that in our simulations, care was taken to make input spectra nonzero so as to ensure the effectiveness of the proposed algorithm.

5. CONCLUSION

We have presented an FFT/IFFT based blind SIMO channel identification method. With a null guard interval inserted in the input, the proposed algorithm is efficacious for a very small size of received data, for which the existing methods become ineffective. This is practically useful when high mobility occurs in wireless channels. Furthermore, with no assumption of the precise knowledge of channel order, the new method identifies both channel parameters and channel order.

References


