

ML Time Domain Channel Estimation and Equalization for OFDM Without Guard Interval

Gagik Mkrtchyan, Katsuhiko Naito, Kazuo Mori and Hideo Kobayashi

Department of Electrical and Electronic Engineering, Mie University, Japan
gagik@com.elec.mie-u.ac.jp, naito@elec.mie-u.ac.jp, kmori@elec.mie-u.ac.jp, koba@elec.mie-u.ac.jp.

ABSTRACT

Proposed Maximum Likelihood (ML) time domain channel estimation and equalization method provides a simple, robust scheme for compensation of the Inter-Symbol Interference (ISI) and Inter-Carrier Interference (ICI) in OFDM system without Guard Interval (GI) using Scattered Pilots (SP). The method employs direct estimation of channel's impulse response, which compared with other methods that use frequency domain estimation, does not suffer from ICI. Also it does not use FFT, which for case of OFDM without GI actually becomes the source of ICI, and requires additional ICI cancellation. Simulation results show that proposed scheme can achieve higher bandwidth efficiency and lower Bit Error Rate (BER) than conventional frequency domain methods.

Keywords: OFDM Without Guard Interval, ISI, ICI, ML Estimation, Bandwidth Efficiency, Scattered Pilots.

1. INTRODUCTION

Guard Interval (GI), as the cyclic extension of data symbol, was inserted into OFDM signal to remove ISI and to assure the simple zero forcing frequency domain equalization. But with all its advantages the OFDM system with GI has following serious weaknesses:

- The maximum delay difference of the channel must be shorter than the length of the GI, otherwise the BER will strongly degrade because of ISI;

- In mobile communication can frequently occur changes of channel's delay difference (for example when user moves from indoor to outdoor environment), so the GI must be chosen based on the longest delay difference case, which decreases the bandwidth efficiency, because to the each symbol must be added GI with long duration;

- The transmitter spends additional power for GI, which in the receiver will be discarded.

For compensation of mentioned disadvantages of OFDM system were proposed multiple methods of equalization and interference cancellation for cases with insufficient GI or without GI [1]-[5]. When OFDM system does not use GI or GI has insufficient length in addition to the ISI arises another problem – subcarriers lose orthogonality and appears the ICI. As a result the methods that employ frequency domain channel estimation and equalization must use various schemes of

ICI cancellation. The main problem of all frequency domain methods is that they always suffer from ICI, but actually the ICI is the side effect of DFT (FFT) usage in the receiver, and the ICI cancellation is additional unnecessary operation.

If to think about DFT not as transformation from the time domain into frequency domain, but as some type of linear transformation, then the ICI cancellation is another linear transformation, and their summary effect is some other third linear transformation. FFT is the optimised version of DFT, which uses less computations – $n \log_2(n)$ instead of the n^2 , but ICI cancellation will generally require additional n^2 computations, so the whole efficiency of using FFT vanishes. For OFDM without GI or with insufficient GI usage of two-step (DFT + ICI cancellation) frequency domain channel estimation and equalization become more computationally inefficient than the direct time domain channel estimation and equalization.

In this paper we discuss the direct (without using of DFT) time domain maximum likelihood (ML) channel estimation and equalization scheme. The proposed method is different from all other previously proposed methods because it is working in the time domain (without FFT). It uses known preamble symbols for ML estimation of channel's impulse response and employs it for detection of current data symbol using already detected previous data symbol to remove the ISI. Because in the case of time domain equalization it is possible that equalization matrix becomes badly conditioned (near to the singular), was used known Scattered Pilots (SP) to compensate the rank reduction of matrix. The power of SP can be equal to the zero (virtual carriers), which allows increasing of power efficiency compared with conventional OFDM with GI. The results of simulation show that proposed scheme has lower BER and can effectively compensate the ISI on case of long delay difference of the channel.

2. COMMUNICATION MODEL

The received signal, which was propagated through the quasistatic multipath-fading channel with Additive White Gaussian Noise (AWGN), can be written as:

$$r(t) = \sum_{l=0}^L h_l \cdot s(t - \tau_l) + w(t) \quad (1)$$

where: $s(t)$ is the transmitted signal;
 τ_l is the delay of l -th propagated path;
 h_l is the channel's impulse response;
 L is the number of delay paths;
 $w(t)$ is the AWGN.

In multicarrier communication system transmitted signal is the composition of the amplitude and phase modulated pure tones:

$$s(t) = \sum_{n=0}^{N-1} A_{nk} \cdot x_n(t) \quad \text{for } kT_s \leq t < (k+1)T_s \quad (2)$$

where: T_s is the duration of symbol;
 N is the total number of subcarriers;
 $x_n(t)$ is the n -th not modulated subcarrier;
 A_{nk} is the data transmitted by n -th carrier on the k -th symbol time.

When the subcarriers, as in OFDM, are periodic ($x_n(t+mT_s) = x_n(t)$) then sampled complex envelope of the received signal will become:

$$r(kT_s + qT_c) = \sum_{n=0}^{N-1} \left[\sum_{m=0}^q A_{nk} \cdot h_{(q-m)} \cdot x_n(mT_c) + \sum_{m=1}^{L-q} A_{n(k-1)} \cdot h_{(q+m)} \cdot x_n((N-m)T_c) \right] + w(kT_s + qT_c) \quad (3)$$

where: T_c is the duration of sample ($T_s = N \cdot T_c$);
it is assumed that the channel's maximum delay difference less than symbol duration;
also in (3) the $h_l = 0$ when $l < 0$ or $l > L$.

For OFDM the $x_n(qT_c) = e^{2\pi i \frac{n}{N} qT_c}$, so (3) becomes:

$$r(kT_s + qT_c) = \sum_{n=0}^{N-1} \left[\sum_{m=0}^q A_{nk} \cdot h_{(q-m)} \cdot e^{2\pi i \frac{n}{N} mT_c} + \sum_{m=1}^{L-q} A_{n(k-1)} \cdot h_{(q+m)} \cdot e^{-2\pi i \frac{n}{N} mT_c} \right] + w(kT_s + qT_c) \quad (4)$$

In (4) the first sum inside the [] is the propagated delayed paths of current k -th symbol and the second sum is the ISI from $(k-1)$ -th symbol.

3. PROPOSED ML DETECTION FOR OFDM

For quasistatic multipath channel (channel's impulse response does not change during one frame) the h_l in (3) can be assumed to be constant, and the $w(kT_s + qT_c)$ are independent Gaussian random variables. Then the ML solution of (3) becomes the standard least mean square (LMS) problem:

$$\arg \min_{\alpha_n} \left(\sum_{q=0}^{Q-1} \left| Y(q) - \sum_{n=0}^{N-1} \alpha_n \cdot Z_n(q) \right|^2 \right) \quad (5)$$

where: $Y(q)$ is the q -th sample of observed signal;
 $Z_n(q)$ are the known functions of q ;
 α_n are N unknown parameters that must be

estimated by minimization of the (5);

the notation is $|X|^2 = X \cdot X^*$.

When α_n are discrete the solution of (5) requires the search between all the possible values of parameters. But for the continuous case the minimization of (5) becomes the solution of system of linear equations:

$$\sum_{n=0}^{N-1} \alpha_n \cdot \left[\sum_{q=0}^{Q-1} Z_n(q) \cdot Z_m^*(q) \right] = \sum_{q=0}^{Q-1} Y(q) \cdot Z_m^*(q) \quad (6)$$

where: m takes all the values from 0 to $(N-1)$.

(6) can be used for estimation of channel's impulse response for OFDM system without GI, using preamble symbol. When the duration of the OFDM symbol is twice longer than the channel's maximum delay difference, to exclude the effect of ISI in the estimation must be used only the received samples $r(kT_s + qT_c)$ for which $q > L$, then in (3) the second sum inside [] will become 0 (which is the ISI) and (3) can be rewritten as:

$$r(kT_s + qT_c) = \sum_{l=0}^L h_l \cdot \sum_{n=0}^{N-1} A_{nk} \cdot x_n((q-l)T_c) + w(kT_s + qT_c) \quad (7)$$

when $q > L$

Then employing (6), the ML estimate of channel's impulse response will become the solution of the system of linear equations:

$$\sum_{l=0}^L h_l \cdot \left[\sum_{q=L+1}^{N-1} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} A_{nk} \cdot A_{nk}^* \cdot x_{n_1}(q-l) \cdot x_{n_2}^*(q-m) \right] = \sum_{q=L+1}^{N-1} \sum_{n=0}^{N-1} r(kT_s + qT_c) \cdot A_{nk}^* \cdot x_n^*(q-m) \quad (8)$$

where: m takes all the values from 0 to L ;
the notation $x_n(q)$ corresponds to $x_n(qT_c)$.

For OFDM the $x_n(qT_c) = e^{2\pi i \frac{n}{N} qT_c}$, so (8) becomes:

$$\sum_{l=0}^L h_l \cdot \left[\sum_{q=L+1}^{N-1} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} A_{nk} \cdot A_{nk}^* \cdot e^{2\pi i \frac{n_1}{N} (q-l)T_c} \cdot e^{-2\pi i \frac{n_2}{N} (q-m)T_c} \right] = \sum_{q=L+1}^{N-1} \sum_{n=0}^{N-1} r(kT_s + qT_c) \cdot A_{nk}^* \cdot e^{-2\pi i \frac{n}{N} (q-m)T_c} \quad (9)$$

It can be shown that when ML estimation formula (9) is applied to the OFDM with GI of enough length, and the GI part of received signal is discarded then it will become conventional frequency domain (after FFT) channel estimation.

Strictly speaking (6) can not be used for ML data detection in OFDM (and any other digital communication system) because the transmitted data symbols are discrete and have limited quantization step, which means that ML detection requires the search through the all-possible values of the data constellation using minimization of the (5).

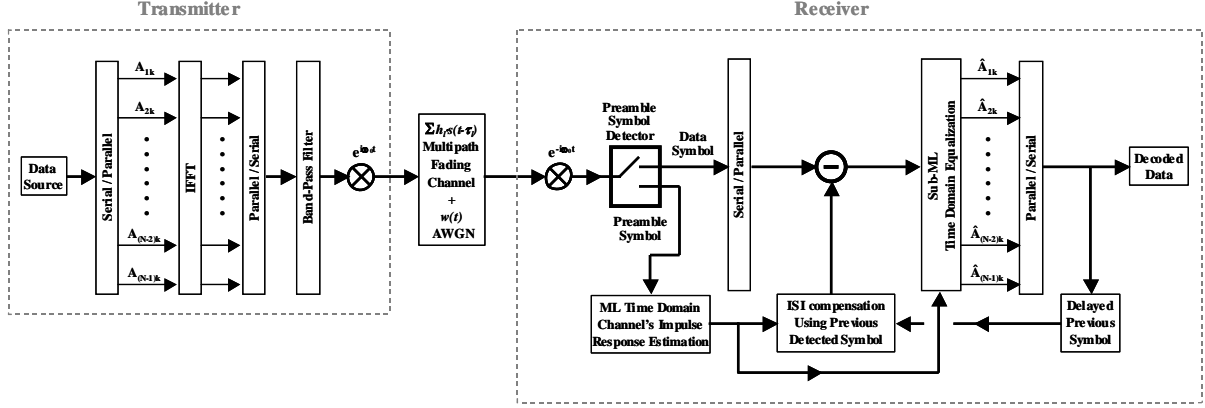


Fig. 1: Functional scheme of proposed time domain ML channel estimation and equalization for OFDM without GI.

But (5) in the N dimensional space of parameters α_n represents the convex paraboloid, which has single global extremum point (because here all the coefficients of second order polynomial are positive it is always point of minimum), so when the noise power is small compared with signal power then the ML discrete solution of (5) is the point nearest to the ML continuous solution of (6). It can be found as the quantization of continuous solution in the domain of the data points constellation. The data detection realized by this way is the sub-ML (but not ML) solution of (5) using (6) and can be written as the solution of the system of linear equations:

$$\sum_{n=0}^{N-1} A_{nk} \cdot \left[\sum_{q=0}^{N-1} \sum_{l_1=0}^q \sum_{l_2=0}^q h_{(q-l_1)} \cdot h_{(q-l_2)}^* \cdot x_n(l_1 T_c) \cdot x_m^*(l_2 T_c) \right] = \sum_{q=0}^{N-1} \sum_{l_2=0}^q \left[\left(r(kT_s + qT_c) - \sum_{n=0}^{N-1} \sum_{s=1}^{L-q} A_{n(k-1)} \cdot h_{(q+s)} \cdot x_n((N-s)T_c) \right) \cdot h_{(q-l_2)}^* \cdot x_m^*(l_2 T_c) \right] \quad (10)$$

where: m takes all the values from 0 to $N-1$;
the notation $x_n(q)$ corresponds to $x_n(qT_c)$.

For OFDM the $x_n(qT_c) = e^{2\pi i \frac{n}{N} q T_c}$, so (10) becomes:

$$\sum_{n=0}^{N-1} A_{nk} \cdot \left[\sum_{q=0}^{N-1} \sum_{l_1=0}^q \sum_{l_2=0}^q h_{(q-l_1)} \cdot h_{(q-l_2)}^* \cdot e^{2\pi i \frac{n}{N} l_1 T_c} \cdot e^{-2\pi i \frac{m}{N} l_2 T_c} \right] = \sum_{q=0}^{N-1} \sum_{l_2=0}^q \left[\left(r(kT_s + qT_c) - \sum_{n=0}^{N-1} \sum_{s=1}^{L-q} A_{n(k-1)} \cdot h_{(q+s)} \cdot e^{-2\pi i \frac{n}{N} s T_c} \right) \cdot h_{(q-l_2)}^* \cdot e^{-2\pi i \frac{m}{N} l_2 T_c} \right] \quad (11)$$

(10) and (11) employ the sequential detection: using of previously detected data symbols for cancellation of ISI and then time domain sub-ML detection of the current data symbol. Similarly to the case of channel's impulse response estimation it can be shown that when sub-ML estimation formula (11) is applied to the OFDM with GI

of enough length and the GI part is discarded, then it will become conventional frequency domain (after FFT) Zero Forcing (ZF) frequency domain detection.

It is possible to think that in sequential detection of OFDM symbols will happen serious error propagation (removing the ISI using the incorrectly detected previous data symbol). But because the number of detection errors is small (especially for high SNR) the time domain signal of previous symbol comes nearly uncorrupted. Actually the sequential time domain detection employs the redundancy of the data in the time domain, which conventional OFDM do not use (data points of GI are discarded) so it's BER performance becomes much better than for conventional OFDM.

4. SIMULATION RESULTS

The performance of proposed ML time domain channel estimation and equalization method was evaluated using computer simulation. The functional scheme of OFDM system with proposed ML time domain channel estimation and equalization is shown in the Fig. 1. Table 1 shows the parameters of simulation. In simulation was used the multipath quasistatic Rayleigh fading channel model with uniformly distributed power profile. Fig. 2 shows the dependence of the BER from the maximum delay difference of the channel for detection using proposed method compared with conventional OFDM. It can be mentioned that proposed method is much more robust against long delay difference of channel, which means that it can provide higher bandwidth efficiency compared with conventional OFDM. Another much more interesting result is that the BER for proposed detection method is about 10 times smaller than for conventional OFDM (even for longer channel differences), as was told in section 3 this happens because the proposed method employs the redundancy of signal. In conventional OFDM the redundancy, given in the time domain as a cyclic prefix of GI, is discarded to remove the ISI, but in proposed method the redundancy is given in the frequency domain as virtual SP, which are employed by

sub-ML time domain equalization to compensate the ICI and increase the BER performance. Figs. 3 and 4 show the dependence of BER on SNR for sufficient GI length case ($L=16$) and insufficient GI length case ($L=20$) in both of cases proposed method provides much smaller BER than conventional OFDM and hasn't the error floor on high values of SNR.

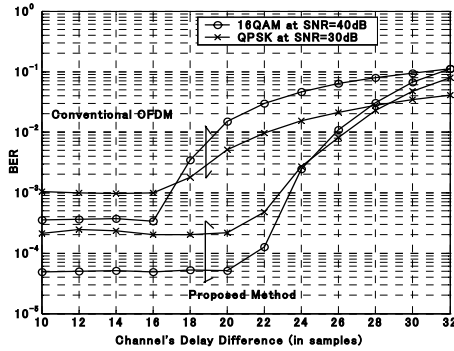


Fig. 2: BER versus channel's maximum delay difference for conventional OFDM with GI=16 and proposed method with SP=16.

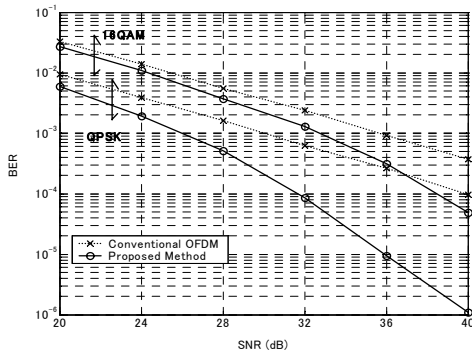


Fig 3. BER versus SNR for conventional OFDM GI=16 and proposed method SP=16. (channel's maximum delay difference is 16).

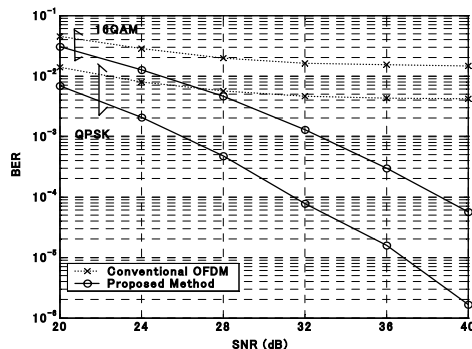


Fig. 4: BER versus SNR for conventional OFDM GI=16 and proposed method SP=16. (channel's maximum delay difference is 20).

Table 1. Simulation parameters.

Number of IFFT points	128
Modulation technique	QPSK, 16QAM
Bandwidth of signal	100MHz
Symbol Duration (convent. OFDM)	1.44 μ s
Symbol Duration (proposed method)	1.28 μ s
Duration of GI (convent. OFDM)	160ns
Number of SP (proposed method)	16
Model of channel	Rayleigh
Number of propagated paths	10-32
Duration of delay difference	100-320ns
Path's power distribution	Uniform
Signal to noise power ratio (SNR)	20-40dB

5. CONCLUSIONS

Proposed ML time domain channel estimation and equalization method is robust against long delay difference of the multipath fading channel and hasn't error floor. It is different from all other previously proposed methods because it is working in the time domain (without FFT). Even though its complexity is higher than the complexity of conventional OFDM if channel's delay difference is less than GI but it provides lower BER compared with conventional OFDM. When the channel's delay difference is longer than the duration of GI the complexity of the conventional OFDM becomes higher than the proposed method, because it requires additional ICI cancellation to compensate the errors of frequency domain channel estimation and equalization.

References

- [1] T. Karp, M.J. Wolf, S. Trautmann, N.J. Fliege, "Zero-forcing frequency domain equalization for DMT systems with insufficient guard interval", in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, April 2003, Hong Kong, China, vol. 4, pp. IV- 221-4.
- [2] S. Suyama, M. Ito, H. Suzuki, K. Fukawa, "A scattered pilot OFDM receiver with equalization for multipath environments with delay difference greater than guard interval," *IEICE Transactions on Communication*, January 2003, vol. E86-B, pp. 275-282.
- [3] S. Trautmann, N.J. Fliege, "A new equalizer for multi tone systems without guard time," *IEEE Communications Letters*, January 2002, vol. 6, pp. 34-36.
- [4] S. Trautmann, N.J. Fliege, "Perfect equalization for DMT systems without guard interval," *IEEE Journal on Selected Areas in Communications*, June 2002, vol. 20, pp. 987-996.
- [5] S. Barbarossa, A. Scaglione, G.B. Giarmakis, "Performance analysis of a deterministic channel estimator for block transmission systems with null guard intervals," *IEEE Transactions on Signal Processing*, March 2002, vol. 50, pp. 684-695.