Abstract—The number of people who can be reached through the Internet has been growing rapidly during the last decade. At the same time also the Internet connections have become faster. Especially this is true at universities. As more and more students with fast Internet connections can be reached, the incentive to use this new possibility to share courses between universities increases.

This paper presents a successful method for delivering mathematical modelling courses and contents through the Internet. The possible future of the mathematical modelling experience is also discussed.

Index Terms—Education, Internet technology and applications, Modelling and simulation

I. INTRODUCTION

Mathematical modelling is a teaching discipline which is extraordinarily suitable to be handled through the Internet. There are various reasons for this. The main reason is that mathematical modelling is such a wide discipline that it basically covers all the areas of mathematics partially including physics and computer science as well. Furthermore, it is also needed in all areas of engineering. This means that the teacher of mathematical modelling should be able to handle a wide variety of different areas. In reality it is obvious that it would be better to have many experts from the different areas of applied mathematics, computer science etc., instead of having just one person to teach mathematical modelling. The problem that arises from this seemingly good approach is how to bring these experts together to hold one course. As solution for this is to merge their expertise by using the Internet, so that each expert represents their own expertise in the learning environment. The learning environment on the other hand should be created with the guidance of the pedagogical principles of distance, collaborative and problem-based learning. Furthermore, the use of several persons responsible for only their own specialized area makes it possible to keep the information in the environment easily up to date.

During 2001, a group of Finnish mathematicians representing different fields of mathematics decided to provide mathematical modelling in the form of distance education for all the Finnish universities that were willing to join this experiment. The main reason was that, so far, there had not been any common studying entity for this field of mathematics at Finnish universities, and the Internet seemed to offer an obvious solution in this situation. As a result, we now have a functioning Web environment for the studying of mathematical modelling in the form of distance education. Distance education offers many benefits, such as access to a broader student audience, a better medium for meeting the needs of students, cost savings and, sometimes and importantly, the use of the principles of modern learning pedagogy [1]. During the last decade, significant political and public interest has been shown towards distance education [2]. Our mathematical modelling platform aims to provide a consistent system for the teaching of modelling subjects.

In this paper, we briefly describe how our hypermedia-based modelling experiment was carried out. A more complete description of the used techniques etc. can be found in [3]. We also discuss the future of the mathematical modelling environment that has been delivered to the Internet.

II. MATHEMATICAL MODELLING

The goal of mathematical modelling is to help in understanding the world in which we live. In mathematical modelling the language of mathematics is used to describe the phenomena that we are interested in. Surprisingly many problems from a wide variety of areas can be described in the language of mathematics. Furthermore, since computers are based on the use of two-valued logic that is the backbone of mathematics, all the problems from mathematical modelling can be translated into computers. Computers, on the other hand, are becoming more effective all the time. All in all, mathematical modelling is nowadays perhaps the most important branch of engineering sciences, which can be applied in just about everything.

Although problems may require very different methods of solution, figure 1 outlines a general approach to the mathematical modelling process.

![Fig. 1. Cycle of the modelling procedure, [4]](image-url)

The upper arrow in figure 1 describes the initial part of the modelling process and it is normally the most demanding,
since it involves the replacement of the real world phenomena with the abstract model. The right downward arrow represents the solving of some resulting well-defined mathematical problem. The subsequent solution is usually in a mathematical form, and must in turn be interpreted back into the original real world setting. Finally, in the left upward arrow the solution is checked against the original reality, that is, the validity of the model must be checked. It can be seen that mathematical modelling is a kind of iterative process and it can be described as in figure 2.

![Diagram](image)

**Fig. 2. Iterative nature of the modelling procedure, [5]**

For example, to improve the design of a product, the interdisciplinary modelling team first lists all the requirements the product has to meet. The team also uses data from experiments or tests to find simplifying assumptions and translate the physical phenomena into equations. Then, the derived equations are analyzed, solved, and the qualitative or numerical solution is interpreted. The simplifying assumptions and the computed solution are validated against the data. If the data and the computed solution are not a good fit, some of the simplifying assumptions are relaxed and the model is improved. The analysis, solution process, validation, and relaxation of the simplifying assumptions-cycle is repeated until there is a good agreement between the solution of the model and the data (note that "good agreement" is a rather vague statement and the team will decide what it means, based on the problem considered and the experience of the specialists in the field.) With the more powerful computers, modelling teams have been able to tackle more complex problems, develop more accurate models, get answers in less time, and reduce research and development costs.

The process of mathematical modelling is very close to the normal scientific process:

1) Make observations of the phenomena,
2) Formulate a hypothesis,
3) Use or develop some method to test your hypothesis,
4) Collect some data,
5) Test your hypothesis against the collected data,
6) Confirmation or denial of hypothesis.

In mathematical modelling the hypothesis is often replaced by a mathematical model. We see that mathematical modelling is not only essential for applications but also in some sense the very core of science.

### III. Mathematical Modelling in Virtual Reality

The main baselines for the initiation of the virtual mathematical modelling course project are listed below:

1. Mathematical modelling and simulation are essential methods for research whenever mathematics is to be applied.
2. From this, it follows that it is of vital importance for a person who is studying applied mathematics or working in the area of research and development in industry, to master the methods of mathematical modelling.
3. There has been no comprehensive study environment for mathematical modelling in Finnish universities [6].

The Finnish Ministry of Education has provided the funding for this project. The funds are shared among the participating universities mostly on the basis of how much material they produce for the Web pages of the course. The project coordinator is Tampere University of Technology (TUT) and the other participants are the Rolfin University of Applied Sciences (HUT), the Centre for Mathematical and Computational Modelling at University of Jyväskylä, the Laboratory of Applied Mathematics at Lappeenranta University of Technology (LUT), the Department of Mathematics Sciences at University of Oulu, the Department of Mathematics at Tampere University of Technology (TUT), the Department of Mathematical Sciences at University of Tampere, the Systems Analysis Laboratory at Helsinki University of Technology (HUT), the Finnish IT Centre for Science (CSC) and the Department of Applied Physics at University of Kuopio.

The modelling environment that has been offered through WWW contains the following items [3]:

- Basic and advanced courses in mathematical modelling,
- Students’ projects,
- Tailor-made courses on modelling,
- Support courses on modelling,
- Modelling competitions.

#### A. General Scheme of the Virtual Modelling Courses

Since mathematical modelling is an interdisciplinary subject, we first make the students form groups of 1-3 persons. During the course they work all the time with their group. This kind of collaborative learning seems to be highly suitable for the teaching of mathematical modelling. It also develops the students’ interpersonal skills, which are important in their future working life. The students quite quickly start to take different roles in their groups. For example one takes care of mathematical formulations, one does the computer implementations and one can collect and revise the information. If they can not do their own parts of the modelling then the group has to find the solution together. So they have to learn how to share their knowledge and expertise in order to be able to pass the course.

At each university, we have a contact person for the students, who will help them if they have any problems in doing their weekly exercises, final work, with computers, with programs, in contacting the lecturers etc.

At the beginning of the course we have an opening ceremony, which is implemented in multipoint video conference...
technique (MCU) [3]. All the participating universities can watch it at the same time in their own premises. This is done mainly to give the course a clear starting point, but it also offers a change to introduce course arrangements. At the end of the course we have also arranged a multipoint videoconference, where the student groups present their solutions to their final work and discuss and comment the other groups’ final work. For the rest of the course the students learn more or less according to their own selected timetable.

After the opening ceremony the student groups start to work on their weekly exercises, giving weekly comments on other groups’ solutions and at the end on doing their final assignments. Every week we have a different subject in our course and the weekly lecturer will give a new set of problems, to which the students have to give solutions in one week. The weekly lectures have been videotaped beforehand and delivered to the modelling environment. The students can then watch the lectures from the Internet in their own time (see illustration of an Internet lecture in figure 3). The lectures are combined with the lecture notes using a technique called SMIL (Synchronized Multimedia Integration Language) [3]. The students can choose whether they want to watch the lecturer in a synchronized mode with slides, just the lecturer or go through the slides. They can also watch just certain parts of the lectures if they wish. We have been using a discussion board in the studying environment, so that the students have been able to send their questions directly to the weekly lecturers. These questions are also seen by the other students, and normally students themselves suggest the solutions for the questions asked in the discussion board. The students can also ask questions directly from the contact person at their university.

In order to guarantee the students’ privacy we have used a course delivery platform called AO [7] (see figure 4). The name comes from the Finnish words ’Avoin Opimisympäristö’, which means ‘Open Learning Environment’. In order to get their weekly assignments into the AO the student needs to obtain a user name and password, which are given when he/she registers as a user of the platform. When the students have delivered their solutions to the weekly exercises into AO, there is a certain time at every week when all the answers from the different groups become open for everyone to see. At this point the student groups start to evaluate and compare the answers that the different groups have delivered to the platform. We have given each group one group whose answers they have to comment on, and the groups will deliver their comments to the platform. Finally the weekly lecturer evaluates the weekly exercises and the level of the commentary, and sometimes also gives model answers.

At the end of the course, the student groups have to present their final assignment in a video conference that is arranged simultaneously between all the universities, and here again, the students are expected to comment on the work of other students. At the end, all the student assessments are combined with weights, and the students receive their final grades.

B. Basic Course in Mathematical Modelling

Our basic course in mathematical modelling in spring 2004 included the following lectures with exercises and additional material:

- Mathematics and its applications
- Different types mathematical models
- Basics of mathematical models
- Numerical and programming tools for mathematical modelling
- Differential equations and system theory
- On discrete models and methods
- Approximations and scales
- Decision analysis
- Probabilistic models
- Models and data

Selectable final assignments were the following:
- Modelling of the composting process
- Creation of a ticket buying system
- Finger prints
C. Advanced Courses in Mathematical Modelling

Our advanced courses in mathematical modelling are at this point divided to four modules. In future there will be six or more modules. These modules deal each with one entity of mathematical models. This guarantees a good uniformity of the advanced courses and gives the student a clear categorization for mathematical modelling. The advanced part consists of the following studying modules in 2004-2005:

1) Partial differential equations in mathematical modelling: Introduction, FEM with modelling and simulation of the building of the silicon crystal, Mathematical modelling of the congealing matter with open boundaries, Multiphysics, Acoustics

2) Data analysis in mathematical modelling: Many valued logic - fuzzy logic, Data mining, Conclusion and negotiation analysis, Evolutionary algorithms, Neural networks

3) Statistical models: Introduction, Statistical analysis of the models, Statistical models in controlling of the felling machine, Pattern recognition, Regression, Response-surface models

4) Randomness in mathematical modelling: Introduction, Applications (Financing, Black-Scholes, Telecommunications), Stochastic differential equations (ITO integral), Time series analysis (ARMA-models), Bayesian statistic inversion and MCMC, MCMC and Assimilation (Kalman filter)

IV. CONCLUSIONS AND FUTURE PLANS

The main benefits obtained with this distance education platform have been:

1. Student satisfaction; more than 50% of the students who have taken our modelling course would rather choose this kind of course again than a traditional course.

2. We now have a working teaching entity, which can be used with little effort in the teaching of mathematical modelling.

The mathematical modelling environment that has been produced, can easily be used as compensatory or associating material for modelling. If properly used, it will make it easy to design diverse courses on mathematical modelling. The material is easily combinable for example with material of practical use of mathematics or as additional material to the ordinary mathematical courses for example in statistics, differential equations, discrete mathematics or optimization.

The studying materials that we are preparing at the moment concern the areas of 3D-mathematics, discrete mathematics and optimization.

In future our aim is to connect our mathematical modelling network to other networks, and we are also initiating international co-operation. Some steps towards international co-operation have already been taken in the form of delivering and promoting the creation of English materials into the virtual modelling environment and sponsoring an international summer school in mathematical modelling (summer 2004).

Many studies have shown that cognitive factors, such as learning, performance and achievement in distance education classes are comparable to those observed in traditional classes [8], [9], [10], [11] and [12]. It seems that distance education is becoming a vital part of higher education.

REFERENCES


