A New Geometric Reasoning Technique for Hot Spot Prediction in Sand Casting

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ABSTRACT
In this paper, we report the use of medial axes transformation technique to predict temperature profile and hot spots in solidifying castings. In essence, we proposed a very simplistic yet comprehensive temperature interpolation algorithm which can solve the solidification problem qualitative and quantitatively. The method has the advantages of both the geometric reasoning technique and the conventional numerical simulation. We illustrate the feasibility of the proposed temperature interpolation technique by comparing its solutions with both pure geometric reasoning technique and the finite element method.

Keywords: Medial Axis Transformation, Casting, Hot-Spot Prediction

1. BACKGROUND
Blum introduced the medial axes transformation (MAT) of a 2D region in the 1960s as an aid to describe the biological shapes [1-4]. In two-dimension the medial axis is the locus of the centre of an inscribed disc of maximal diameter as it rolls within the domain by maintaining the contact with the domain boundary, as shown in Figure 1. A collection of medial axes/surfaces and its inscribed circles/spheres are called medial objects.

![Fig.1: A Three-Dimensional Object Showing its Medial Surfaces and Inscribed Sphere [5]](image)

The review of existing literature reveals that medial objects was mostly applied for geometry interrogation and constructions of three-dimensional objects like human body and biological entities possessing very odd shapes. Direct utilization of medial axes in engineering problem is far less and rare. However, noticeable progress in this area was made by Armstrong and co-workers in a series of publications [1-4], in which the medial objects are treated as substituted equivalent structures for massive engineering structural analysis e.g. offshore platform that requires expensive computational effort and time. The work by Armstrong and his colleagues demonstrated that medial objects have great potentials in cutting down dramatically the simulation time and cost in practical engineering application. Motivated by this, this paper proposes to explore the usage of medial objects in casting industry.

A careful study in the casting literature shows that the use of geometric methods in predicting hot spot in solidifying casting is not new. The most widely used technique is probably the so-called Modulus Method or the Chvorinov rule [6], which states that the total freezing time \( t_f \) in the casting of volume \( V \) obeys the following equation of proportionality:

\[
 t_f = B \left( \frac{V}{A} \right)^\lambda ,
\]

where \( B \) is a constant for given metal and mould condition and \( A \) is the surface area available for cooling. An immediate consequence of Eq. (1) is that the freezing time is a single-valued function of \( B \) and this does not allow one to predict the transient temperature profile in the casting. The research based on the Chvorinov rule has been widely investigated [7,8]. Wlodawer [9] documented the use of Chvorinov rule to design the feeder, which will ensure adequate feeding. Neises et al. [10] suggested to compute the moduli of 2D sections using a generalized equation based upon an area/perimeter value for the section modulus, a method, which in essence, is described by equation (1). For a three-dimensional casting, these authors suggested the use of circles, squares, polygon etc instead of using the complete casting volume. Ravi and Srinivasan [11] projected rays from a given point inside a section in all directions and the intersections of the rays with the domain boundary were computed. They concluded that the resultant vector along these rays pointed toward the hotspot. This ray technique assisted in determining the mass concentration center and the effective boundary, which was then used to compute the modulus. It should
be noted that the resultant vector computed by Ravi and Srinivasan [11] was in fact the geometric center rather than the hotspot. Only under the special case of uniform solidification, the geometric center coincides with the hotspot, which was the case investigated by these authors. The same authors [12] also used the direction of the thermal gradient inside a casting to move along a path that leads to the location of hot spot if the thermal gradient is known a priori. A very novel but simplistic method based on section modulus method which can reveal freezing wave in a casting was obtained by Heine and Uicker [13]. From a certain perspective, the method can be viewed as a combination of projected ray method and Chvorinov rule. From this literature review, it was noted that none of the developed method is able to predict transient temperature profile or take into account the sensitivity of the material properties. Moreover, some of the techniques were only applicable to very simplistic geometry while others were cumbersome to implement.

2. HEAT CONDUCTION EQUATION

The governing equations for temperature diffusion in solid can be written as [14-15]:

\[ \nabla \cdot (K \nabla T) + \rho \frac{dH}{dT} \frac{\partial T}{\partial t} - Q = 0 \]  

(2)

where \( T, K, \rho, H, Q \) are temperature, thermal conductivity, material density, specific enthalpy and volumetric heat sink/source, respectively. In order to embed the heat conduction equation into the medial axis, we make the following assumptions: (1) All boundaries of the casting-mould assemblies are homogeneous; (2) heat flux emanates normally from the medial axes of the casting; (3) the mould is relatively thin as compared to the casting. The first assumption follows directly from the Chvorinov’s rule. Assumption (2) and (3) are indeed the constraints for the proposed method. Now, if we ignore the non-linearity in Eq. (1), we notice that the transient temperature solution possesses the self-similar property for a particular case of boundary condition [16]. Following this, we can express the temperature as a function of normalised space and time such that

\[ T = T \left( \frac{x}{L^*}, \frac{t}{t_{\text{max}}} \right) \]  

(3)

Since the medial axis algorithm allows one to find the maximum radius of the inscribed circle, \( R \), by generating an array of arbitrary points with distance \( d_i \) normal to the medial axis, the transient temperature at any arbitrary point in the casting could be found by interpolation using the following equation

\[ T_i = T \left( \frac{d_i}{R} ; \tau = \frac{t}{t_{\text{max}}} \right) \]  

(4)

where \( \xi \) is the normalized radius in the geometry. The actualisation of Eq. (4) in fact possesses some awkward problem. Notice that Eq. (4) represents a three-dimensional curve in \( (T, \xi, \tau) \) space. The temperatures at \( t = t_{\text{max}} \) are unknowns until the solutions at the last time step have been resolved. Therefore to normalise the temperature solution in both space and time, one first requires all the solutions in both space and time. This is however very uneconomic and requires extra programming effort. Bearing in mind that one only requires the qualitative comparison of the temperature, Eq. (4) can be simplified as

\[ T_i = T(\xi, t) \]  

(5)

and the coding now becomes straightforward. Figure 2 schematically depicts the procedure that we adopted.

3. SOME COMPUTATIONAL ASPECTS

The first step in the implementation of the interpolation method is the numerical solution of the one-dimensional heat conduction equation (1). To achieve this, we use the Finite Element Method. For a case involving non-linear solidification, the enthalpy method described in Pao et al. [14] has been implemented. On the element level, the discretised finite element matrix can be given as
where \( n \) is the iteration counter, \( L \) is the local element length, \( A \) is the element area, \( T_t \) and \( T_{t+1} \) are temperature solutions at time level \( t \) and \( t+1 \), respectively and index \( i \) and \( j \) are local node number, respectively. The temperature derivative of the enthalpy is evaluated using the Lemmon’s method \([17]\), which can be written as

\[
\frac{dH}{dT} = \left[ \frac{\partial H}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial H}{\partial y} \frac{\partial T}{\partial y} \right]^{1/2}. \tag{7}
\]

For linear problem, e.g. in the mould domain, one can replace the temperature derivative of the enthalpy, Eq. (7) with the heat capacity of the mould. As for the treatment of the mould-cast interface, we proposed a new method, which we called the heat flux method. Consider Figure 3 where a mould and cast elements are placed side-to-side, with the global node numbered as shown.

**Fig.3:** Mould-Cast Assembly and its Interface Element

The heat flux method makes use of the master-slave element relationship between the cast and the mould. In the first place, the cast element adjacent to the mould is treated as a master. One then proceeds to find the slave (mould) element/node belongs to that master. For the case depicted in Figure 3, for the cast domain, node 3 is the master node with a slave of node 4. Using the Newton’s law of cooling, the matrix equation

\[
\begin{bmatrix}
R_2 \\
R_3
\end{bmatrix} = \begin{bmatrix}
0 \\
h_{ul} T_4
\end{bmatrix}. \tag{8}
\]

is added to Eq. (6) for element 2-3 (the nodal temperature vector is shown here for clarity of the location of the element stiffness in the global position).

The corresponding RHS vector for the master element 2-3 due to the convective nature of slave element 4-5 is given as

\[
\begin{bmatrix}
0 \\
h_{ul} T_4
\end{bmatrix}.
\]

The process is repeated for the mould domain. The advantage of the heat flux method is that it is more flexible when it comes to mesh definition and discontinuous meshes can be used.

4. RESULTS AND DISCUSSIONS

Here we compare qualitative solution from our technique with one of those geometric reasoning method, namely the wavefront method, developed by Heine and Uicker \([13]\) and also with the Finite Element Method. At this point, there is a need to emphasize that only qualitative comparison is possible in our case since the methodology belongs neither to geometric reasoning nor FEM. For some simplistic geometry, quantitative comparison and accuracy of our method with FEM has been carried out in Reference \([15]\) and hence will not be repeated here. Figure 3 shows the solidification wave front predicted using the method in Reference \([13]\) while Figures 4 and 5 show the proposed and the finite-element solution, respectively, at the same time step.

**Fig.4:** Pure Geometric Reasoning Solution by Heine and Uicker \([13]\)

We noticed that the current method predicted the same hot spot location as the wave front method. The wavefront technique, however, can only show non-evolutionary solution while the new technique is able to mimic the whole transient temperature profiles. It is observed that even though our results are not entirely quantitatively equal when compared to the finite element solution (Figures 4 and 5), they are qualitatively similar. The location and size of hotspot (area with maximum temperature in present context) compares favourably between two methods and these are located in the section with the maximum inscribed radius. It is also noted that the temperature contour distribution exhibits similarity for both the solutions, both in the cast and in mould. This new medial-axes based solidification interpolation method is fast and comprehensive and is capable of giving a quicker “feel” of the solution. The interpolated solution is, however, exhibit zig-zag pattern at some location. This is due to the nature of our algorithm, which assumes that the solution at a point is directly influence by one boundary surface only. A smoother temperature solution could be obtained if one assumes that the temperature at a point is influenced by all its adjacent
boundaries but this will increase the computational cost dramatically.

![Interpolated Temperature Solution using the Present Algorithm](image1)

**Fig.5: Interpolated Temperature Solution using the Present Algorithm**

![Solidification temperature solution using the Finite Element Method](image2)

**Fig.6: Solidification temperature solution using the Finite Element Method**

5. CONCLUSION

In this paper, we have presented a new solution algorithm for the prediction of hot spot in casting. The method is a hybrid between the pure geometric reasoning technique and the more conventional finite element method. The result of our analysis showed that the proposed method is favourable when tested with realistic casting problems.

6. REFERENCES


[5] [http://www.fegs.co.uk/medial.html](http://www.fegs.co.uk/medial.html)


