Characterization of Wireless Multi-hop Paths with High Quality: Toward a Theoretical Framework

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ABSTRACT

In this paper, we consider the quality of communication paths in wireless multi-hop networks. A wireless multi-hop network consists of wireless nodes and wireless links. In this network, there usually exist various candidate paths to connect a source and a destination. We can evaluate the quality of each of the candidate paths by using a metric for link quality, and as a result, we can choose the path with the highest quality (best path). In this paper, we pay attention to characteristics of the best path, and consider how to theoretically characterize the best path and its quality because it is an important issue from the viewpoint of network design. We introduce some recent results on theoretical identification of the best path and theoretical formulas to evaluate the quality of the best path in wireless multi-hop networks with regular structure. We explain how the best path is identified using examples. We also introduce results on theoretical characterization of the best path in random wireless multi-hop networks. Finally, we describe some related issues.

Keywords: Wireless multi-hop network, routing, communication quality, theoretical analysis

1. INTRODUCTION

In wireless multi-hop networks [1], each wireless node has capabilities of direct communication with other wireless nodes and relaying to connect distant nodes that cannot be connected directly. These capabilities enable wireless nodes to construct a wireless multi-hop path (route) between two nodes. As a result, a source S can send a packet to a destination D through a wireless multi-hop path even if S and D are not directly connected.

In a wireless multi-hop network, a routing protocol has to be executed to construct a multi-hop path between S and D. There have been a lot of proposals for routing protocols [1]. In ordinary routing protocols, the number of hops is considered as a metric to build a wireless multihop path, and wireless multi-hop paths with minimum hops are usually used.

Other than the number of hops, various metrics have been proposed to construct a multi-hop path of high quality. For example, we have metrics called expected transmission count (ETX) [2], per-hop round trip time (RTT) [3], medium time metric (MTM) [4], and expected transmission time metric (ETT) [5]. These metrics are defined differently for different purposes, and have different properties.

In this paper, we mainly consider ETX because it is a simple metric reflecting the quality of a link. Link ETX is the expected number of packet transmissions required to successfully deliver a packet over the link, including retransmissions. If we compute link ETX assuming a MICA2 mote, which is a typical node for wireless sensor networks, as a wireless node, we can build a model of the link ETX as shown in Fig. 1. In this model, the link ETX is represented as a function with respect to the length of the link. To derive this function, a model of transmission and some parameters shown in [2] and [6] are used. In this example, it is assumed that non-coherent FSK is used as modulation, NRZ encoding is used, the output power is -8 dBm, and noise floor is -105 dBm.

![Fig. 1: ETX function of a link.](image-url)

We can also use ETX to evaluate the quality of wireless multi-hop paths. We evaluate quality of a wireless multi-hop path by the mean value of the total number of packet transmissions required to successfully deliver a packet from S to D over the multi-hop path. This mean value is called route ETX. A wireless multi-hop path usually consists of multiple links.
Then, route ETX of a wireless multi-hop path is computed as the sum of link ETXs of all links included in the multi-hop path.

From Fig. 1, we can see that link ETX increases as the length of a link increases in the above model of ETX. This property can be intuitively understood because transmission errors tend to occur more frequently if we use longer links. In addition to this fact, the lengths of links tend to be long if we use a routing protocol that builds wireless multi-hop paths in minimum-hop fashion. Therefore, we can easily expect that minimum-hop routing increases the link ETX of each link included in a multi-hop path. Conversely, if we use a routing protocol that selects a multi-hop path minimizing the lengths of links, the ETX of each link can be minimized; however, the number of hops increases, and as a result, the route ETX may increase because of the increase of the number of hops.

The above observation indicates that a wireless multi-hop path is desired to have appropriately short links and an appropriately small number of hops to minimize route ETX. It is not difficult to build such a multi-hop path in actual situations if routing protocol can utilize link ETX of each link as a link cost. Actually, link ETX can be measured at wireless nodes and can be distributed over the network. After exchanging information on adjacent nodes and link ETX, nodes in the network own common information of network topology represented as a weighted graph as shown in Fig. 2, where the weight of a link corresponds to link ETX. Then, routing protocol can compute a multi-hop path with minimum cost (best path) by executing a minimum cost algorithm.

![Fig.2: Model of a multi-hop network.](image)

Considering such a routing, we have interesting problems from the viewpoint of theoretical network design. For a multi-hop network with regular structure, we may be able to theoretically identify the best path by utilizing simplicity of network topology without executing minimum cost algorithms. Furthermore, such a theoretical identification may give properties of the best path in detail. In another case where wireless nodes are randomly distributed, it is also interesting to theoretically characterize the best path and to statistically estimate route ETX from typical parameters that characterize a random network. These theoretical analyses not only give theoretical insights but also contribute to network planning and optimization. For example, solutions to these problems can be applied to optimization of the network parameters to achieve a desired route ETX.

With this as background, in this paper, we introduce some recent results on theoretical identification of the best path and theoretical formulas to evaluate the quality of the best path in wireless multi-hop networks with regular structure [7]. We explain how the best path is identified using examples. We also introduce results on theoretical characterization of the best path in random wireless multi-hop networks [8]. To obtain these results, theories of graph/network and geometrical probability [9] [10] are utilized as mathematical tools. Finally, we describe some related issues.

2. ROUTE ETX IN ONE-DIMENSIONAL NETWORKS

In this section, we use ETX as a metric of link quality, and introduce how the best path is theoretically identified in two cases that are different in assumptions on positions of nodes. In one case (Case 1), nodes are at constant intervals of $a$ on a line, and in another case (Case 2), nodes are randomly distributed on a line obeying a Poisson distribution with intensity $\lambda$. Let $u(z)$ be the link ETX of a link of length $z$. We assume that $u(z)$ is a convex monotonically increasing function, and $u(0) > 0$ as represented in Fig. 1. Let $L$ be the distance between $S$ and $D$. Let $d$ be the maximum transmission range (communication range) of a node. We assume that $u(z) = \infty$ if $z > d$. Let $U(r)$ be the route ETX of path $r$. Let $R_k$ be the set of all paths with $k$ hops. Let $r_{O,k}$ be the best path. Let $r_{O,k}$ be a path that minimizes route ETX in $R_k$. Then, $U(r_{O,k})$ is the minimum route ETX, and is equal to $\min_k U(r_{O,k})$.

2.1 Case 1

In Case 1, we can construct a weighted graph using $a, L, d$, and $u(z)$. In the weighted graph, there is a link between two nodes if the distance between them is not longer than $d$, and the weight of a link is decided by substituting the distance between these nodes into $u(z)$. Figure 3 shows an example. In this example, $a = 8m$, $d = 50m$, $L = 88m$ and the ETX function in Fig. 1 is used. Then, one way to obtain the best path and minimum route ETX is to execute a minimum cost algorithm with the weighted graph as an input. Of course, this method successfully gives the best path and minimum route ETX; however, the minimum cost algorithm only outputs the best path, and does not indicate properties of the best path.
Also, outputs of the algorithm do not give a clear relation between minimum route ETX and \(a, L, d,\) and \(u(z)\). The best path in Case 1 is determined by \(n_{1,k}, \ell_{1,k}, n_{2,k}, \ell_{2,k}, s,\) and \(t\).

Let us observe the above formula using an example. Figure 4 shows an example of the best path between S and D, where \(a = 8m, d = 50m, L = 88m\) and the ETX function in Fig. 1 is used. In the following, we call the routing method that selects the best path Optimum Routing (OR). Namely, OR minimizes route ETX. In this example, after some calculations, we have \(n_0 = 3, s = 3\) and \(t = 4\). This means that the number of hops of the best path is 3 or 4. Namely, the best path is \(r_{O,3}\) or \(r_{O,4}\). Then, we have to consider \(r_{O,3}\) and \(r_{O,4}\) to find the best path. We have \(n_{1,3} = 1, n_{2,3} = 2, \ell_{1,3} = 3a = 24,\) and \(\ell_{2,3} = 4a = 32\); therefore, \(r_{O,3}\) consists of one link of length 24m and two links of length 32m. In the same manner, we can see that \(r_{O,4}\) consists of one link of length 16m and three links of length 24m. Then, from the link costs shown in Fig. 3, route ETX of \(r_{O,3}\) is \(u(24) + 2u(32) = 7.1\), and that of \(r_{O,4}\) is \(u(16) + 3u(24) = 5.6\). From these values, we can see that the best path is \(r_{O,4}\), and the minimum route ETX is 5.6. As can be seen from this example, the best path can be clearly identified, and minimum route ETX can be computed very easily in Case 1. Also, we can clearly know what length each link should be set to and what number of hops should be selected to construct the best path. In this example, the link lengths should be 16m and 24m, and the number of hops should be four.

In Case 1, we showed in [7] that there is a direct relation between the best path and the parameters \(a, L, d,\) and \(u(z)\). Namely, we have a method to successfully identify the best path without executing any minimum cost algorithms owing to simplicity of the network structure. The following are the results in [7]: In this method, we do not have to construct any weighted graphs for inputs, and inputs are just \(a, L, d,\) and \(u(z)\). In Case 1, minimum route ETX is given as

\[
U(r_{OR}) = \begin{cases} 
U(r_{O,1}), & L \leq n_0a, \\
\min_{k} U(r_{O,k}), & L > n_0a, 
\end{cases}
\]  

(1)

where \(n_0\) is a positive integer that minimizes \(\frac{u(n_0a)}{n_0a}\).

This equation means that the number of hops of the best path is \(s\) or \(t\). Also, we know that links included in \(r_{O,k}\) are classified into at most two groups. For a given \(k\), let \(n_{1,k}\) and \(\ell_{1,k}\) be the number of links and the length of each link in the first group, respectively. Let \(n_{2,k}\) and \(\ell_{2,k}\) be the number of links and the length of each link in the second group, respectively. Then, we have

\[
n_{1,k} = k - n_{2,k},
\]

(2)

\[
n_{2,k} = \frac{L}{a} - k \left[ \frac{L}{ka} \right],
\]

(3)

\[
\ell_{1,k} = \left[ \frac{L}{ka} \right] a,
\]

(4)

\[
\ell_{2,k} = \ell_{1,k} + a.
\]

(5)

In Fig. 5, we show minimum route ETX for different values of \(a\) as functions of \(L\). Here, \(a^{-1}\) means the density of nodes. From Fig. 5, we can see that as the density of nodes increases, the minimum route ETX tends to decrease for a given \(L\). This is because we have more candidate paths as the density increases. Also, as \(L\) increases, the minimum route ETX increases for a given \(a\) because more hops or longer links are needed to construct a multi-hop path with high quality between S and D.

For reference, we consider routing methods called Shortest Path Routing (SPR) and Longest Path Routing (LPR). Figures 6 and 7 show examples of multi-hop paths selected by SPR and LPR, respectively. Parameters used in these figures are the same as those used in Fig. 4. SPR selects a path with minimum hops. In Fig. 6, S and D can be connected by a two-hop path because \(d = 50m\) and \(L = 88m\). The lengths of the first and second links are 48m and 40m, respectively. Then, link ETXs of the first and second links are \(u(48) = 15.4\) and \(u(40) = 6.2\), respectively. Therefore, the route ETX of the multi-hop path se-
lected by SPR is 21.6 in this case. In Fig. 7, LPR is considered. This method selects a multi-hop path consisting of the shortest links. Namely, the multi-hop path in Fig. 7 consists of eleven links of length $\alpha = 8m$. The link ETX of each link is $u(8) = 1.0$. Therefore, the route ETX of the multi-hop path selected by LPR is 11.0 in this case. From the above three examples, we can see that the route ETX depends on how to determine the length of each link and the number of hops, and that it is important to know what length each link should be set to and what number of hops should be selected to minimize the route ETX.

Let us compare the route ETX of OR (minimum route ETX) with those of SPR and LPR in more detail. Figure 8 shows the ratio of the route ETX of SPR to the minimum route ETX, and the ratio of the route ETX of LPR to the minimum route ETX as functions of $L$. In this figure, $\alpha = 5m$. From this figure, we can confirm that for a small $L$, the ratios are close to 1, namely, OR is not so different from SPR and LPR. For a large $L$, the ratios are greater than 1. From these results, we can see that OR greatly reduces ETX compared with SPR and LPR, if $L$ is relatively large.

As explained in this section, we can successfully identify the best path in Case 1. The results in [7] enable us to know all links included in the best path, as well as the number of hops in the best path, without executing any minimum cost algorithms like the Dijkstra algorithm. While this result is considered to be interesting from a theoretical point of view, it can be applied to optimization problems of multi-hop networks as well as estimation of the minimum route ETX.

2.2 Case 2

In this subsection, we consider the route ETX of the best path in Case 2, and introduce approximate methods to analyze the minimum route ETX in a one-dimensional random network [8]. If nodes are randomly distributed as assumed in Case 2, inputs for analysis of the route ETX will be

- Density of nodes.
- Distribution that represents statistical properties of positions of nodes.
- Distance between source and destination.
- Communication range.
- Function of link ETX.

Here, we assume that the distribution of nodes obeys a Poisson distribution of intensity $\lambda$. As can be seen from the above list, in Case 2, input is not given as a weighted graph. Furthermore, in this case, we should evaluate the route ETX using the statistical property of the minimum route ETX. Hence, output of the analysis should be a statistical value. Then, we use the mean of the minimum route ETX. In this subsection, we consider how to theoretically compute the mean of the minimum route ETX using the above listed inputs.

In Case 2, it is difficult to precisely compute the mean of the minimum route ETX using theoretical methods, and an approximate method is needed. Then, in [8], we considered a simple routing policy...
to select a multi-hop path, denoted by Policy 1 here. This policy is an approximation of Optimum Routing. After defining Policy 1, we theoretically compute the mean route ETX of Policy 1 using the theory of geometrical probability [9][10]. The simplicity in Policy 1 enables us to theoretically compute the mean route ETX in random multi-hop networks with Policy 1.

The following is Policy 1: Policy 1 tries to choose a multi-hop path with links close to a constant $d_{s,opt}$ and an appropriately small number of hops. Define that $d_{s,opt}$ is a real number that satisfies $u(d_{s,opt}) = 2u(d_{s,opt}/2)$. Then, Policy 1 chooses a link whose length is not longer than $d_{s,opt}$ and is the closest to $d_{s,opt}$, as the next link. If there is no link whose length is not longer than $d_{s,opt}$, then Policy 1 chooses a link whose length is longer than $d_{s,opt}$ and is the closest to $d_{s,opt}$, as the next link.

Figure 9 is an example of route selection based on Policy 1. In this example, S has three nodes within $d_{s,opt}$ and chooses the closest node, denoted by A, in these three nodes as a relay node. Next, node A has no node within $d_{s,opt}$, then node A chooses the closest node, denoted by B, as the next relay node. Node B has three nodes within $d_{s,opt}$ and chooses node C. Because the distance between node C and destination D is not longer than $d_{s,opt}$, they are directly connected. Consequently, Policy 1 chooses the multi-hop path S-A-B-C-D.

In [8], we showed that the mean route ETX of Policy 1 is at most double the mean of the minimum route ETX for a large density of nodes. Also, for a small density of nodes, there are a small number of candidate paths in nature; therefore, it is expected that paths selected by Policy 1 are not so different from the best path, and that the mean route ETX of Policy 1 can approximate the mean of the minimum route ETX. Also, this expectation is supported by the result in Case 1 because the difference between the lengths of links included in the best path is at most $a$ in Case 1.

Let $U_A$ be the mean route ETX of Policy 1. If $L \leq d_{s,opt}$, then $U_A = u(L)$ because source and destination are directly linked by Policy 1. For $L > d_{s,opt}$, we have

$$U_A = \sum_{k=[L/d]}^{2[L/d_{s,opt}] - 1} \Pr(H = k)U_k,$$

where $H$ is a random variable representing the number of hops of a multi-hop path selected by Policy 1, and $U_k$ is the mean route ETX of a multi-hop path selected by Policy 1, given that $H = k$. In [8], $Pr(H = k)$ and $U_k$ were derived by the theory of geometrical probability. This theory can be applied to analysis of properties of geometrical patterns whose positions are random [9][10]. Study of the minimum route ETX using the geometrical probability is a new topic in the field of performance analysis of wireless multi-hop networks. In the same manner, in [8], we derived the mean route ETXs of SPR and LPR in Case 2 using geometrical probability.

We can approximately compute the minimum route ETX in Case 2 for given $\lambda, L, d$, and $u(z)$ using the formula for Policy 1. In Fig. 10, we show some numerical results the mean route ETX of Policy 1 together with the simulation results of the mean of the minimum route ETX, where $d = 50m$, and the ETX function in Fig. 1 is used. This figure shows that the numerical results of the approximate method agree with the simulation results. From this result, we can confirm that Policy 1 well approximates OR in Case 2, and that it is important to construct a multi-hop path consisting of links whose lengths are close to $d_{s,opt}$ to minimize the route ETX. This result indicates that even in random multi-hop networks, we can characterize the best path and can know what kind of path minimizes the route ETX.

Figure 11 shows the ratio of the mean route ETX of SPR to the mean of the minimum route ETX, and the ratio of the mean route ETX of LPR to the mean of the minimum route ETX as functions of $L$. In this figure, $\lambda = 0.2$. For the theoretical values, the means of route ETXs of SPR and LPR are obtained by equations in [8]. Figure 11 shows that OR significantly reduces route ETX compared with SPR and LPR if $L$ is relatively large in the same manner as in Case 1.
3. ROUTE ETX IN TWO-DIMENSIONAL NETWORKS

In the preceding section, we briefly introduced theoretical characterizations of the best path in two cases in one-dimensional multi-hop networks. Of course, the topology of multi-hop networks is not limited to one-dimension. The above results should be extended to the same problems in two-dimensional multi-hop networks.

In [7], a lattice network shown in Fig. 12 was considered. The inputs to computation are as follows.

- Distance between adjacent nodes: $a$.
- Position of source node: $(0,0)$.
- Position of destination node: $(L_x, L_y)$.
- Communication range: $d$.
- Function of link ETX: $u(z)$.

To theoretically identify the best path, the following assumptions are made in [7].

- $u(z)$ is a convex monotonically increasing function.
- $1 < \frac{u''(z)}{u'(z)} < 5$ for $0 \leq z \leq d$, where $u'(z)$ and $u''(z)$ are the first and second derivatives of $u(z)$, respectively.
- $u(d) > 2\left(\frac{d}{\sqrt{2}}\right)$.

The last two assumptions are made to simplify the identification of the best path. Again, let $r_{a,k}$ be a multi-hop path that minimizes route ETX in $R_k$.

Let $n_x$ and $n_y$ be $\frac{L_x}{ka}$ and $\frac{L_y}{ka}$, respectively. We define $n_{3,k}, n_{4,k}, n_{5,k}$ and $n_{6,k}$ as follows:

$$n_{3,k} = k - n_{4,k} - n_{5,k} - n_{6,k},$$  \hspace{1cm} (7)

$$n_{4,k} = \frac{L_x}{a} - km_x - n_{6,k},$$  \hspace{1cm} (8)

Let $r_{2,k}$ be a path consisting of $n_{3,k}$ links, $n_{4,k}$ links, $n_{5,k}$ links and $n_{6,k}$ links represented as vectors $(m_xa, m_ya)$, $(m_xa + m_ya)$, $(m_xa, m_ya + a)$, and $(m_xa + a, m_ya + a)$, respectively. The following are the results in [7]: Under the above assumptions, it was shown that $r_{a,k} = r_{2,k}$ if $U(r_{2,k}) < \infty$. Also, if $L_x > 0$ or $L_y > 0$, we have

$$U(r_{OR}) = \min_{1 \leq k \leq L_x, L_y} U(r_{2,k}).$$  \hspace{1cm} (11)

From this equation, we can compute the minimum route ETX. Also, the best path can be characterized by $r_{2,k}$. The best path in the lattice network can be theoretically identified although some restrictions on the link ETX function are needed.

An example of the best path is shown in Fig. 12, where $a = 8m$, $d = 50m$, $(L_x, L_y) = (96, 32)$ m, and the ETX function in Fig. 13 is used. In this example, the number of hops in the best path is five. Let us observe what kinds of links are included in the best path. In this case, $m_x = 2$ and $m_y = 0$. Then, links included in the best path are represented by four vectors $(2a, 0) = (16, 0)$, $(3a, 0) = (24, 0)$, $(2a, a) = (16, 8)$, and $(3a, a) = (24, 8)$. Also, we have $n_{3,5} = 0$, $n_{4,5} = 1$, $n_{5,5} = 3$ and $n_{6,5} = 1$. Therefore, $(16, 0)$ is not included in the best path, and the best path consists of a vector of $(24, 0)$, three vectors of $(16, 8)$ and a vector of $(24, 8)$. As a result, the best path consists of three kinds of links as shown in Fig. 12. Link ETXs of vectors $(24, 0)$, $(16, 8)$ and $(24, 8)$ are 1.5, 1.1 and 1.6, respectively. Therefore, minimum route ETX is 6.4. Note that although we have to compare $U(r_{2,k})$ for different $k$ in Eq. (11) to obtain the above best path, the range of $k$ for comparisons is 3 ≤ $k$ ≤ 16 in this case.

In Fig. 14, we show the minimum route ETX for different destinations in a lattice network, where the source node is at $(0,0)$, $a = 8m$, $d = 50m$, and the ETX function in Fig. 13 is used. In this figure, route ETX at $(x, y)$ is the minimum route ETX for a destination at $(x, y)$.

4. RELATED ISSUES

Thus far, we have introduced how to characterize the best path and how to theoretically compute the minimum route ETX in three cases. In this section, we consider other issues related to characterization of the best path in wireless multi-hop networks. As mentioned, in some cases, we can know the direct
relations between typical parameters of the network and the quality of the best path; however, these results are obtained only for limited cases, and there will be many cases in which similar analyses are possible. For example, we have the same problem for a random two-dimensional network. Furthermore, we have regular structures other than line structure and lattice structure. We have distributions of nodes other than a Poisson distribution. It is important to consider the same problems for these structures.

Also, in this paper, we introduced just analyses of route ETX. As mentioned in the introduction, we have other metrics for link quality, and analyses of path quality for these metrics should be also studied. In particular, we utilized an assumption that link ETX is a convex monotonically increasing function. Analysis of path quality for the metrics that do not satisfy this assumption is more difficult than that of ETX. For example, if we use medium time metric (MTM) [4] as cost of a link, MTM is no longer a convex monotonically increasing function because it is defined as the medium time consumed in a link in a multi-rate environment as represented in Fig. 15. Hence, the results in Secs. 2 and 3 are no longer applicable to the networks with MTM. In addition to analysis of MTM, analyses of metrics other than ETX and MTM will be challenging research issues.

The results introduced in this paper can be applied to other kinds of problems. We have a lot of network problems. For example, we have load balancing, construction of disjoint paths, and channel assignment. These issues can be considered together with communication quality. For example, we can consider balancing a load on each node while guaranteeing path quality. For this purpose, the results explained in this paper can be utilized because we can know what kind of paths achieves the highest quality. From this information, we can obtain candidate of paths that achieve the highest quality, and can consider load balancing using these candidates.

In this paper, we assumed static nodes and paid attention to communication quality of a path. We should also analyze effects of mobility and interference on communication quality.

5. CONCLUSIONS

In this paper, we considered quality of communication paths in wireless multi-hop networks, and introduced some results on identification and characterization of the best path. Using the results, we could precisely identify the best path and estimate the minimum route ETX in regular multi-hop networks. In a random multi-hop network, although some kinds of approximations are needed, we have some approximate methods that successfully described the characteristics of the minimum route ETX. We also considered some extensions and applications of these results. We still face many other interesting problems related to characterization of the best path as sum-
6. ACKNOWLEDGMENT

The authors would like to thank Mr. Kazuyuki Miyakita of Graduate School of Science and Technology, Niigata University for his kind help.

References


