A new efficient algorithm for real time harmonics measurement in power systems

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ABSTRACT

The paper introduces a new LS algorithm for a real time measurement technique that can track the harmonics in power system waveforms. The proposed algorithm does not require matrix inversion. The technique produces accurate real time measurements of the amplitude and phase of the harmonics present in a power system. To verify the theoretical analysis, computer simulation has been used and tested with a hypothetical signal. The results obtained confirm that the method is computationally efficient and suitable for real time applications. The simulation has shown that the computational requirement has been reduced significantly and that the identification of harmonics buried in a noisy signal is accurate.

Keywords: Power systems harmonics, LS algorithm.

1. INTRODUCTION

Power systems are often subject to harmonic distortion due to increasing applications of nonlinear loads. The existence of harmonics in power systems could cause serious problems such as voltage distortion, increased losses and heating, and malfunction of protective equipment. The presence of voltage and current waveform distortion is generally expressed in terms of harmonic frequencies that are integral multiples of the power system nominal frequency. Power system harmonics have become one of the important indexes rating power quality issues. Good power quality means less distortion and less harmonics in the voltage and current sources.

The design of harmonic filters relies on the accurate measurement of harmonic distortion in both current and voltage waveforms in real time [1-2]. There are many different approaches to the measurement of harmonics, such as FFT, application of adaptive filters and neural networks [3-4]. However, most of them can only operate effectively in a narrow range of frequencies, at moderate noise levels and often require prior knowledge of the number of harmonics present in the system.

A method of solving the system equations without matrix inversion using an application of the singular value decomposition (SVD) has been presented in [5]. It enables accurate measurements of the amplitude and phase angle of each of the harmonics present in a power system. However, the matrices were complex which is a concern for real time application.

This paper presents a new algorithm which involves only real numbers and can solve the system equations without matrix-inversion. The algorithm can calculate the amplitudes and phase angle of harmonics by simply multiplying each set of input signals by a constant matrix. The technique is suitable for use in real time applications. The method is capable of measuring the harmonic signals accurately.

To verify the theoretical analysis, computer simulation has been used and tested with a hypothetical signal. The result obtained proves that the method is accurate and computationally efficient and therefore suitable for real time applications.

2. ALGORITHM REALISATION

2.1 Review of linear least squares algorithm

A time varying waveform of voltage or current in a power system of known fundamental angular frequency, \( \omega_0 \), with harmonics of unknown magnitudes and phases can be expressed as:

\[
y(t) = \sum_{i=1}^{k} \alpha_i \cos(\omega_0 t + \phi_i) + e(t)
\]  

(1)

In which \( \alpha_i \), \( \omega_0 \) and \( \phi_i \) are the unknown amplitude, frequency and phase angle of the \( i^{th} \) harmonic. The variable \( e(t) \) represents the additive Gaussian noise with unity variance. The signal can be written in terms of complex sinusoids \( \alpha_i e^{j(\omega_0 t + \phi_i)} \). Thus from (1),

\[
y(t) = \frac{1}{2} \sum_{i=1}^{k} \alpha_i \cos(\omega_0 t + \phi_i)
\]

\[
= \frac{1}{2} \sum_{i=1}^{k} \left( \alpha_i e^{j\phi_i} e^{-j\omega_0 t} + \alpha_i e^{-j\phi_i} e^{j\omega_0 t} \right)
\]  

(2)
Next, consider the set of \( l \) number of samples \( \alpha_i \), \( \alpha_2 \ldots \alpha_{n+l} \) of the waveform. The number of measurements \( n \) is typically higher than the number of harmonics, \( k \), which is \( l > 2k \). The discrete-time version of (2) can be written in matrix notation as

\[
y = AB
\]

(3)

Where

\[
y = \begin{bmatrix} y(n) \\ y(n-1) \\ \vdots \\ y(n-l) \end{bmatrix}, \quad B = \begin{bmatrix} \alpha_i e^{j\phi} \\ \alpha_k e^{j\phi} \\ \vdots \\ \alpha_l e^{j\phi} \end{bmatrix}
\]

and

\[
A = \frac{1}{2} \begin{bmatrix} e^{j\omega_1} & e^{j\omega_{2}} & \cdots & e^{j\omega_{k}} \\ e^{j\omega_{2}} & e^{j\omega_{3}} & \cdots & e^{j\omega_{k+1}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\omega_{l}} & e^{j\omega_{l+1}} & \cdots & e^{j\omega_{n}} \end{bmatrix}
\]

\[
\Phi = \begin{bmatrix} e^{-j\omega_1 T} & 0 & \cdots & 0 & 0 \\ 0 & e^{-j\omega_1 T} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & e^{-j\omega_{k} T} & 0 \\ 0 & 0 & \cdots & 0 & e^{-j\omega_{n} T} \end{bmatrix}
\]

The significance of (5) is that matrix \( P \) is a constant matrix and \( \Phi \) is a time varying matrix. Note that \( \Phi \) is square and zero for non-diagonal elements. Substituting for \( A \) from (5) into (4),

\[
B = (A' A)^{-1} A' y = \left( (P \Phi)^{T} P \Phi \right)^{-1} (P \Phi)^{T} y
\]

(6)

Let \( C = (P \Phi)^{T} P \Phi \), \( C \) is a constant matrix and \( \Phi^{-1} = \Phi^{*} \) (complex conjugate). In this way the complexity of the vectoring operations in (4) can be reduced as follows:

\[
B = C \Phi^{*} y
\]

(7)

It can be noted that, the elements of matrix \( P \) are complex exponentials of the form \( e^{j\omega_{i} T} \) and can be represented in rotation matrix form with real numbers, by letting the matrix \( H_{m}^{n} \) be an equivalent representation of \( e^{-j(\omega_{i} T)m} \), where \( m = l \) and \( i \) is the harmonic order. The new \( P \) matrix is then:

\[
P = \begin{bmatrix}
H_{0}^{0} & H_{1}^{0} & \cdots & H_{k}^{0} \\
H_{1}^{-1} & H_{1}^{1} & \cdots & H_{k}^{1} \\
H_{2}^{-1} & H_{2}^{1} & \cdots & H_{k}^{2} \\
\vdots & \vdots & \ddots & \vdots \\
H_{l-1}^{-1} & H_{l}^{1} & \cdots & H_{k}^{l} \\
\end{bmatrix}
\]

(8)

where

\[
H_{m}^{n} = \begin{bmatrix}
\cos \omega_{n} T m & -\sin \omega_{n} T m \\
\sin \omega_{n} T m & \cos \omega_{n} T m \\
\end{bmatrix}
\]

The amplitude and phase angle for each harmonic component can thus be obtained.

\[
B = \begin{bmatrix}
b_{a1} \\ b_{b1} \\ \vdots \\ b_{ak} \\
\end{bmatrix} = \begin{bmatrix}
\alpha_{1} \cos \phi_{1} \\ \alpha_{1} \sin \phi_{1} \\ \vdots \\ \alpha_{k} \cos \phi_{k} \\ \alpha_{k} \sin \phi_{k} \\
\end{bmatrix}
\]

(9)

The amplitude \( \alpha_{k} \) and phase angle \( \phi_{k} \) of the \( k^{th} \) order signal harmonic is thus given by
\[ \alpha_i = \begin{cases} \left| b_{Ri} \right| \left[ 1 + \left( \frac{b_{Ri}}{b_{Ri}} \right)^2 \right], & \left| b_{Ri} \right| > \left| b_{Ri} \right| \\ \left| b_{Ri} \right| \left[ 1 + \left( \frac{b_{Ri}}{b_{Ri}} \right)^2 \right], & \left| b_{Ri} \right| < \left| b_{Ri} \right| \end{cases} \]

\[ \phi_i = \tan^{-1} \left( \frac{b_{Ri}}{b_{Ri}} \right) \]  \hspace{1cm} (10)

As may be seen from (7), the mathematically complex and thereby time consuming matrices equation (3) can be solved by using the linear least squares method without the need to invert a matrix. It is extremely efficient computationally, since it only performs one matrix multiplication per sample time. The size of the matrix is $2k \times l$ and therefore $2k \times l$ multiplication and addition operations are required.

3. SIMULATION RESULTS

The periodic non-sinusoidal waveforms are described in terms of their harmonics, whose magnitudes and phase angles are computed using Fourier analysis. The analysis permits a periodic distorted waveform to be decomposed into an infinite series. Positive and negative half cycles of power system voltages and currents tend to have identical shapes so that their Fourier series contain only odd harmonics. Simulations were conducted to demonstrate the technique of determining harmonic components using the new algorithm. The simulated signal is

\[ y(t) = \begin{cases} 100 \cos \omega t + 60^\circ & + 60 \cos 3\omega t + 45^\circ \\ +50 \cos 5\omega t + 36^\circ & + 40 \cos 7\omega t + 30^\circ \\ +30 \cos 9\omega t + 20^\circ & + 20 \cos 11\omega t + 10^\circ \\ +10 \omega 13\omega t - 10^\circ & + e(t) \\ \end{cases} \]

$0 \leq t \leq 0.40 \text{sec}$

\[ 100 \cos \omega t + 60^\circ & + 70 \cos 3\omega t + 30^\circ \\ +60 \cos 5\omega t + 20^\circ & + 50 \cos 7\omega t + 10^\circ \\ +40 \cos 9\omega t + 36^\circ & + 30 \cos 11\omega t + 20^\circ \\ +5 \cos 13\omega t - 90^\circ & + e(t) \\ 0.40 \text{sec} \leq t \leq 0.80 \text{sec} \]  \hspace{1cm} (11)

where \( \omega = 314 \text{ rad/s} \),

\[ e(t) = \text{white noise of zero mean unity variance.} \]

The signal waveform contains the basic component at 50Hz and the 3rd, 5th, 7th, 9th, 11th and 13th harmonics. The sampling frequency is 3.5 kHz. The number of samples \( l=30 \) and \( n=100 \).

To investigate the real time ability of the proposed approach the signal condition described by (11) was simulated.

Figures 1 and 2 show the simulation results. It is clear that the new technique is able to identified the amplitude and phase angle for each harmonic of the time-varying signal accurately.

Noticeably, the new technique produces fast and accurate tracking of harmonic components as evident from the step change which occurs at 0.4sec.
Fig. 3 compares the original hypothetical signal, as defined in (11), and the reconstructed signal using the new LMS algorithm. The reconstructed signal is the same as the original signal in the steady state i.e. after about 50 msec. The initialization process results in a mismatch in phase information. However, after this initialization time, any variation is tracked rapidly and accurately with only a delay of half a cycle signal period as shown in Fig. 3. This technique is rapid and accurate and is suitable for use in real time applications.

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![Graph](image)

**Fig. 4:** Comparison of the proposed method with conventional method for identifying the fundamental, 3rd and 7th harmonic amplitudes.

The proposed method guarantees very low error in the amplitude and phase values of each harmonic component in a power system waveform.

![Graph](image)

**Fig. 5:** Comparison of the proposed method with conventional method for identifying the fundamental, 3rd and 7th harmonic phase angles.

**4. CONCLUSIONS**

This paper presents a new algorithm for the measurement of amplitude and phase angles of time varying harmonics based on LS without using matrix inversion. The new method is appropriate for tracking harmonics with time-varying amplitude and phase angles. Simulation results show the excellent accuracy and minimal response time for the new technique which is easy to understand and implement compared with traditional methods of harmonic tracking.

**5. REFERENCES**


