Water Level Control for Stabilizing Platform

Pichet Suebsaiprom*, Saroj Saimek†, and Ake Chaisawadi‡

Department of Control System and Instrumentation Engineering
e-mail: s6403505@st.kmutt.ac.th* and ake.cha@kmutt.ac.th‡

Department of Mechanical Engineering
e-mail: saroj@fibo.kmutt.ac.th†

King Mongkut’s University of Technology Thonburi, Thailand

ABSTRACT

This paper proposes nonlinear and linearized dynamic models of a platform balancing system using water level control. The system consists of four balancing bladder tanks placed on corners of the platform. The platform has two degrees of freedom (2-DOF), pitching \( \theta \) and rolling \( \phi \). Control objective is achieved by stabilizing as well as tracking control of the platform. Integral state feedback (ISF) control law is utilized for the linearized model. Simulations demonstrate both open-loop and closed-loop control strategies. Both nonlinear and linearized model behaviors are compared by open-loop simulation. The closed-loop system, without stabilizing mass, demonstrates zero steady state error.

Keywords: Fish Robot, Stabilizing Platform

1. INTRODUCTION

Underwater robots are widely used in many applications; for example, deep-sea exploration, military operations etc. For implementation in high performance, they can operate in limited and critical environment. Fish robot investigations are interested. There are many problems involved in the fish robots. One of the problems is its system stability such as orientation and stationary stabilizing for precise manipulation of underwater object. In this paper, we concentrate on orientation stabilizing of the fish robot. Generally, fish utilize their swimbladder as a stabilizer, which is working by varying internal air volume for control its buoyancy balance. Thus, in order to achieve fish robot stability, we proposed a mechanism called platform stabilizing, which is similar to swimbladder operation in fish.

The balancing bladder tank is set on the reference platform inside the robot. Fig.1 shows how to apply the platform balancing to the fish robot. In general, motion of fish body has 6-DOF has three translations (x, y, and z) and three rotations (roll, pitch, and yaw). The three rotations cannot be directly controlled when the robot is in buoyancy balancing mode, therefore we propose a balancing platform to stabilize two of the three rotations, which are pitch \( \theta \) and roll \( \phi \) rotation. By using water level control for stabilizing the platform. The concept of balance beam using water level control was proposed [4]. It can rotate freely in 1-DOF, after that some researchers [1], [5], [6] used balance beam case to study in nonlinear dynamic system control by the intelligent control techniques such as neuro-fuzzy and fuzzy transition logic control. Shields et.al. [2] proposes detailed mathematical model, controller design, and experimental results of the balancing beam. They succeed in controlling a 1-DOF system. This paper extends the concept in order to stabilizing platform by derived its mathematical model in 2-DOF.

![Fig. 1: Motion of fish robot](image)

The rest of the paper is presented as follows. Section 2 explains the system description, derived nonlinear and linearized mathematical model of water level control for stabilizing platform. Section 3 presents a controller design for closed-loop control system. Section 4 shows simulation results in open-loop and closed-loop control. Section 5 provides concluding remarks.

2. DYNAMIC MODELING

2.1 System Description

The system consists of four balancing bladder tanks placed on four corners of the platform. The platform can freely rotates about its center (o) in 2-DOF as shown in Fig. 2. Meaning that it rotates about x and y-axis. Pitch angle is controlled by different water level \( \Delta h_\theta \) between bow and stern tank on the platform. Similarly, the system’s roll angle can be controlled by different water level \( \Delta h_\phi \) between left and right tank. Both difference levels are controlled by two pairs of bidirectional water pumps. First pair is pump 1 and pump 3, which can pump water from tank 1 (right bow) to
tank 3 (right stern) and tank 2 (left bow) to tank 4 (left stern) respectively. The same action can also be used to change the difference water level $\Delta h_\theta((h_1+h_2)-(h_3+h_4))$. A pair of similar to the first pair, pump 2 and pump 4 can pump water from tank 1 to tank 2 and tank 3 to tank 4 respectively. The same action can also change the difference water level $\Delta h_\phi((h_1+h_3)-(h_2+h_4))$. All water pumps are reversible and have the same characteristics. The system as above is called the hydraulic platform balancing system.

![Fig. 2: Structure of platform balancing system](image)

### 2.2 Nonlinear Model

The dynamic modeling of the platform balancing system can be derived by using the Lagrange’s equation.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = F_i \quad i = 1, 2 \ldots n \quad (1)$$

Where $F_i$ and $q_i$ are generalized force and coordinate, respectively. $L$ is the difference between kinetic ($T(q_i, \dot{q}_i)$) and potential energy ($V(q_i)$) which.

$$L(q_i, \dot{q}_i) = T(q_i, \dot{q}_i) - V(q_i) \quad (2)$$

An equivalent model is shown in Fig. 3. $m_1, m_2, m_3$, and $m_4$ represent mass in tank 1, tank 2, tank 3, and tank 4, respectively. $m_p$ represents structure mass such as pump and aluminum frame, and $m_s$ represents stabilizing mass. The stabilizing mass makes the system open-loop stable and facilitates to identity system’s parameters in experimental and it will be removed later.

We can transform the mass at positions ($x_{ij} = \pm w$, $y_{ij} = \pm l$, and $z_{ij} = h/2$) into positions in base reference frame ($x_{0j}$, $y_{0j}$, and $z_{0j}$) by using transformation matrix.

$$\begin{bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \sin \theta & \cos \phi \sin \theta \\ 0 & \cos \theta & -\sin \theta \\ -\sin \phi & \sin \phi \cos \theta & \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} x_{0j} \\ y_{0j} \\ z_{0j} \end{bmatrix} \quad (3)$$

The kinetic and potential energy is given as.

$$T(q_i, \dot{q}_i) = \frac{1}{2} \sum_{i=1}^{n} (m_i \dot{x}_{ij}^2 + \dot{y}_{ij}^2 + \dot{z}_{ij}^2) \quad (4)$$

$$V(q_i) = \sum_{i=1}^{n} m_i g z_{ij} \quad (5)$$

Fig. 3: Equivalent model of platform balance

Substitute the energy equations into Eq. (2) and solve the Lagrange’s equation, thus the nonlinear dynamic model of the pitch angle is.

$$\ddot{\theta} = \frac{\tau_\theta(\Delta h_\theta) \cos \theta - h_\theta \dot{\theta} - h_\phi(\Delta h_\theta) \dot{\phi}}{J_\theta(\Delta h_\theta)} \quad (6)$$

The nonlinear dynamic model of the roll angle (neglected pitch angular acceleration ($\ddot{\theta}$) effect) is given by.

$$\ddot{\phi} = \frac{\tau_\phi(\Delta h_\phi) \cos \phi + \dot{\phi} J_\phi(\Delta h_\phi) \dot{\phi} - h_\phi(\Delta h_\phi) \dot{\phi}}{J_\phi(\Delta h_\phi)} \quad (7)$$

Subsystem of the stabilizing platform is composed of the difference water level in each pair of tanks and the water pumps. The water pumps can pump water from one tank to another tank, which cause changing in water level and its reversibility gives the difference water level in a pair of bow-stern ($\Delta h_\theta$) and left-right ($\Delta h_\phi$) tanks. Both subsystems can be estimated by the first order linear dynamical models.

$$\Delta h_\theta = \frac{2Q_1}{A}, \quad \Delta h_\phi = \frac{2Q_2}{A} \quad (8), (9)$$

$$\dot{Q}_i = -\frac{1}{\tau_p}(Q_i + K_p u_i) \quad (10)$$

$$\dot{Q}_2 = -\frac{1}{\tau_p}(Q_2 + K_p u_2) \quad (11)$$

where $R_\theta$ and $R_\phi$ = damping coefficients of platform

$\tau_\theta(\Delta h_\theta) = \text{torque due to } \Delta h_\theta$

$\tau_\phi(\Delta h_\phi) = \text{torque due to } \Delta h_\phi$

$J_\theta(\Delta h_\theta) = \text{rotational moment of inertia}$
$J_\phi(\Theta, \Delta \dot{\phi})$ is rotational moment of inertia of the platform. $Q_1$ and $Q_2$ are the flow rate of water in the tanks. $A$ is the area of each tank. $K_\tau$ is the motor constant of the pump. $\tau_p$ is the time constant of the motor. $u_1$ and $u_2$ are the output of the controller (voltage) and remain term are mechanical damper.

2.3 Linearized Model

Although, Eq. (6) and (7) represent the platform in a wide range of operations, they are quite complex. However, we require the system to operate in only slightly deviation of pitch and roll from their equilibrium point. Therefore, the linearized system is considered. To linearize the system, we require the system to operate in only slightly deviation of the error comparator and the plant as shown in Fig. 4. The new matrix expression is given by $\dot{x} = Ax + Bu$ and $y = Cx$.

We then insert two integrators state $z_1$ and $z_2$ in the forward path between the error comparator and the plant as shown in Fig. 4. Therefore the system equations, Eq. (20) and Eq. (21), can be augmented. The new matrix expression is given by

$$\begin{bmatrix} \dot{x} \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} A_{x} & 0_{x,2} & 0_{x,2} \\ -C_{z,1} & 0_{z,1} & 0_{z,1} \end{bmatrix} \begin{bmatrix} x \\ z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} B_{u} & 0_{u,2} & 0_{u,2} \\ 0_{u,1} & 0_{u,1} & 0_{u,1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0_{x,2} \\ 0_{z,1} \end{bmatrix} \phi_{\text{ref}}$$

(22)

We found that the Eq. (22) is controllable, in order to place dominant poles to the closed-loop system, we consider characteristic equation $\lambda I - A + B_k K_c = 0$. The $K_c$ is state feedback gain. The control input is given as

$$u = -K_c x$$

(23)

Fig. 4: Block diagram of ISF controller

The dominant and other placed poles of closed-loop system depend on the system responding to achieve the specific goal. The state feedback gain achieved by utilizing MATLAB function $K_c = \text{place}(A, B_k, \text{desire poles})$ and demonstrate desire pole $\approx [-1.72 \pm 1.2]$, $-50 -50 -51 -52$. The four complex conjugate poles are closed loop dominant pole with $\zeta = 0.8$, $\omega_n = 2.15 \text{ rad/sec.}$, and desired outputs transient response are $t_s = 4 \text{ sec.}$ and $\% \text{OS} = 1.1\%$.

4. SIMULATION RESULTS

In order to demonstrate the system’s response, we specified the system as follows, length $l$ equals 24.0 cm, width $w$ equals 9.0 cm, area of each tank $A$ equals 95.03 cm$^2$, $H$ equals 10 cm, and $m_g$ equals 3 kg. Fig. 5 shows open-loop simulations compared between the nonlinear and linearized models. Therefore, utilizing nonlinear and linearized models for small pitch and rolling should not different system’s responses. According to expect, in Fig. 5 we found that, open-loop responses of both nonlinear and linearized system demonstrate similar behavior.

Fig. 5: Simulation response of open-loop models
The output angles of system can be tracking both reference angles and give zero steady state error to non-zero reference angle ($\theta_{\text{ref}}$ and $\phi_{\text{ref}} \neq 0$) with $t_s = 4$ sec., $\%\text{OS} = 1.65\%$ for $\theta$ output and $t_s = 4$ sec., $\%\text{OS} = 1.85\%$ for $\phi$ output. Although in simulation, transient response of the system can be arbitrary provided by dominant stable poles, in experimental it should be careful due to the limitation of the system’s internal states such as maximum flow rate, maximum sum of $\Delta h$, etc. Fig. 8 shown the platform controlled simulation in 3D animation movement.

5. CONCLUSION

Output angles of reference platform provided by $\Delta h_\theta$ and $\Delta h_\phi$. The pitch angle ($\theta$) is controlled by different water level $\Delta h_\theta$ and the roll ($\phi$) angle controlled by different water level $\Delta h_\phi$. Practically, the total different water level is depended on total water level in each pair ($\Delta h_\theta + \Delta h_\phi \leq H$). The simulation results show the system behavior, controllability, stabilizability, and zero steady state error in closed-loop ISF control. Their control strategy will be implemented and applied to stabilizing the fish robot later.

6. REFERENCES


APPENDIX

Refer to equation (6).

$$
\tau_x(\theta, \Delta h_\theta) = 6 \rho m \Delta h_\theta^2 + 2 \rho m \theta^3 + 32 \rho m \theta + 16 n \theta + 16 n \theta
$$

$$
\tau_y(\phi, \Delta h_\phi) = 32 \rho m \theta \Delta h_\phi \cos \theta + 8 \rho m \Delta h_\phi \sin \theta
$$

$$
g_x(\theta, \Delta h_\theta) = 16 \rho m \theta \Delta h_\theta \cos 2 \theta + 3 \rho m \Delta h_\phi \sin 2 \theta
$$

Refer to equation (7).

$$
\tau_x(\phi, \Delta h_\phi) = (3 \rho m \Delta h_\phi^2 + \rho m \phi^3 + 16 \rho m \phi + 16 \rho m \phi)
$$

$$
+ (16 n \phi + 16 m \phi) \sin \theta
$$

$$
\tau_y(\phi, \Delta h_\phi) = 32 \rho m \theta \Delta h_\phi \sec \theta \cos \phi + 8 \rho m \Delta h_\phi \sin \phi
$$

$$
+ 16 \rho m \phi + 16 n \phi + 16 m \phi \sin \phi
$$

$$
g_x(\theta, \Delta h_\theta) = (6 \rho m \Delta h_\theta^2 + 2 \rho m \theta^3 + 32 \rho m \theta
$$

$$
+ 16 n \theta + 16 n \theta) \sin 2 \theta
$$

$$
g_x(\theta, \Delta h_\phi) = 16 \rho m \theta \Delta h_\phi \cos \theta
$$